About Quantum Field Theory

Dave Peterson, 3/15/20 - 10/5/22 *Preliminary*.

The basic "substance" of physical reality is the superposition of the various fields that constitute the omni-present "Vacuum" of space-time: what Wilczek calls "the Grid" or "Core." This includes the quantum fields of the "Standard Model" of particle physics along with metric gravity, dark energy and condensates such as "Higgs." Real or "borrowed" energy in the Core can "<u>create</u>" (excite) the known particles which can then be annihilated (de-excited). The following is not the physics or math of Core quantum field theories [QFT] but rather what they are "about." We first look at some comprehension barriers to learning, then how QFT compares to QM, and how all quantum theories live in spaces that require the Born Rule of "squaring" to re-enter our reality.

Stumbling Blocks to Learning Quantum Field Theory:

Quantum Field Theory [QFT] is "arguably the most successful scientific theory of all time." The "Standard Model" [SM] of particle physics is a quantum field theory. But, QFTs are challenging to learn, not easy to understand, hard even to define, and very difficult to interpret. Some aspects of these difficulties include:

QFTs aren't really "field" theories in the usual sense of assigning values to points in space-time *(…real or complex scalar values, vectors, tensors).* Instead, they assign "operators" to each space-time point; and these "operators represent the whole spectrum of possible values so that they rather have the status of observables [Teller, 1995]. Also, space-time geometrical 'points' are too small and have to be broadened with a local smoothing function such as a small bell-shaped profile. That could imply that there is really a deeper higher-energy theory below the standard model that we need to presently avoid because we have no idea how to deal with it. We "cut off" sizes that are too small {"effective field theory"}.

QFT also cannot be said to be all about particles or all about fields but some broader meaning of "quanta" that includes these; neither particles nor fields "takes precedence over the other [Teller]." An essential attribute of being a "particle" is being localized in space. But, the "Reeh-Schlieder Theorem" of 1961 says that a particle cannot be completely localized inside any box no matter how big. And entanglement between space-like separated points is unavoidable in QFT. Also, "field quanta are not well defined in the presence of interaction" [Auyang]; *and we do care about interactions!* <u>"When energy is delivered to an interaction event, a "bubbling cauldron" of outcome possibilities are evoked from the 'Vacuum'</u> [Lancaster]. All of these possibilities are superimposed with each other.

"Weinberg's recent QFT textbook explicitly begins with particles and constructs the fields as auxiliary objects. However, the general consensus in QFT appears to be that the subject is primarily about quantum fields. In fact, much modern research in the field only really makes sense from this viewpoint" [Wallace-E]. "Of course, none of this is to deny that particles exist, merely that they are not part of the fundamental ontology of quantum field theory."

Remember that "in quantum mechanics we do not actually describe particles. We describe the probabilities for measurement results."

It is often stated that QFT is the successful merging of quantum mechanics and special relativity (QM & SR). But that description also fits "relativistic quantum mechanics" {RQM, which was not yet a quantum field theory – although it can be formulated with QFT operators on particles already in existence}. It is also important to note that "one can "formulate nonrelativistic QFT, and that's what condensed-matter physicists do nowadays all the time. Phonons are the vibrations of the crystal lattice, and in the QFT description they appear as particle-like excitations" [Forums-n].

Quantum theory in total includes the usual non-relativistic quantum mechanics (QM=NRQM), RQM -- such as Dirac theory and Klein-Gordon theory, quantum electrodynamics (QED), electro-weak field theory (EWT), quantum chromo-dynamics (QCD), and other QFTs. These are Lagrangian *action* theories, and the "Standard Model" of particle physics is itself a Lagrangian QFT (LQFT). Lagrangians in classical mechanics are just an alternative and equivalent formulation of Newton's mechanics. In QM, they make sense as "phase counters" – how many wavelengths has one gone through along a path. Then preferred path end-points are those with greatest wave-phase reinforcements or constructive interference.

"In spite of overwhelming success in particle physics and condensed matter physics, QFT itself lacks a formal mathematical foundation. For example, according to "Haag's theorem," there does not exist a well-defined interaction picture for QFT. This in turn implies that perturbation theory of QFT, which underlies the entire Feynman diagram method, is fundamentally ill-defined" {even though we are able to make it work for us very well}.

Attempts to make QFTs more mathematically intelligible include "Algebraic QFT" [e.g., Halvorson]. AQFT "is currently the most promising proposed axiomatization of QFT" [Frasier]. This endeavor begins with various sets of logically fundamental axioms that define free particles. But development after that has evolved slowly – troubles still appear when particle interactions occur. In part, this again may be due to a different deep reality existing below current field theories. "The major problem with AQFT is that very few concrete theories have been found which satisfy the AQFT axioms. To be precise, the only known theories which do satisfy the axioms are interaction-free."

A leading theorist said, "Quantum field theories are by far the most complicated objects in mathematics, to the point where mathematicians have no idea how to make sense of them [Tong]." "Quantum field theory is mathematics that has not yet been invented by mathematicians."

Due to the smallness of its electromagnetic coupling "constant," α, QED can be a perturbation theory to different powers of α. But higher order terms in the calculations have long been plagued with infinities that were finally systematically subdued by "renormalizing" electron charge and mass, e and m_e. All viable quantum theories have to be renormalizable. The "renormalization group" [RG] looks at force laws versus scaling of energy/momentum. As a technicality, "the renormalization group is not really a group as re-scaling and cuttoffs {Λ s} are not invertible" – there is no inverse transformation. RG tells how the scale Λ changes with the maximum momentum of interest. And, if we indeed summed the entire renormalized perturbation series in QED, it would be divergent!

As one example of scaling, "LEP accelerator experiments in 2002 showed that the fine structure 'constant' of QED was measured to be about $1/127$ at energies close to 200 GeV, as opposed to the standard low-energy physics value of $\alpha \simeq 1/137$ " {-- the perceived value of electric "charge" scales with energy}.

QFT compared to usual quantum mechanics, QM:

QM and RQM apply to particles that are already in existence -- they are unable to change particle number by creating photons or new massive particles. That is a major purpose of the fields of QFT. In both QM and QFT, states and observables are equally important. We still have Hilbert spaces and superpositions of states. In QM, an electron is a particle that has wave aspects; but in QFT, an electron is a field that can have normal mode excitations. The "collapse of the wave function" remains an enigma in both formalisms [stack]. In simple terms collapse represents an unexplained discontinuity where, in order to describe what we observe, we jump from one theoretical model (the wave) to another (the particle).

We are used to the familiar QM having position states in configuration space often labeled by the letter "psi," ψ(x,t). "It is important to realize that the operator valued field ϕ(x,t) or $\hat{\phi}$ in QFT is not analogous to the wavefunction ψ(x,t) in QM." While ψ in QM is acted upon by observables/operators, "in QFT it is the (operator valued) field itself which acts on the space of states" [Plato]. In the usual "Schrodinger picture," states are functions of time and operators are not. But the alternative "Heisenberg picture" of QM has states as constant and operators are functions of time. It is this picture that most easily generalizes to QFT.

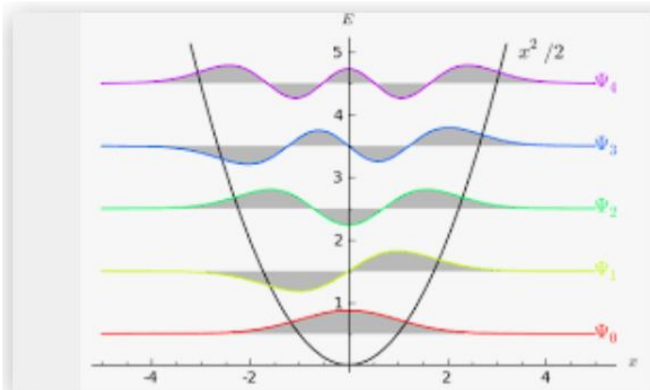
A typical "state" in QFT can specify the momentum or spin state of a particle, {|k,s⟩, in Dirac "ket" notation}. *{ $k = 2\pi/\lambda$ is vector-wavenumber so that $p=\hbar k = h/\lambda$.* Note: it is convenient in modern physics to let speed of light be one, $c \equiv 1$ and also $h/2\pi \equiv \hbar = 1$. Then p = k and E = ω }.

A major convention in QFT is that "momentum states" are built up from the fundamental "vacuum state" |0⟩ using creation operators: $\hat{a}^\dagger_k |0\rangle = |k\rangle$ where operator $\hat{a}^\dagger_k$ {"a-dagger"} "increases by one quanta the number count in normal mode k" [Weinberg]. "The mathematics for the creation and annihilation operators for bosons is the same as for the ladder operators of the quantum harmonic oscillator" [Wik].

> *Recall that in linear algebra, for a matrix operator a, the notation $a^\dagger$ means the "adjoint" or "Hermitian conjugate" or transpose of the complex conjugate of a i.e., $a^\dagger=(a^*)^T$. In QFT, we go one step further for "overbars" $\bar{\psi} = \psi^\dagger\gamma^o$ on Dirac "spinors" where gamma-zero is a gamma matrix. Since that involves transposes (interchange rows and columns), a Dirac column spinor becomes a row spinor.*

"The multiparticle states in a quantum field theory, at least at the perturbative level, are described by a "Fock" number space" {e.g., $\psi = |n_{k1}, n_{k2}, \ldots n_{ki}\rangle$ where n_{ki} is the occupation number of momentum state $|k_i\rangle$ "Two types of fields are distinguished: matter fields and interaction fields with quanta of fermions and bosons (half-integral spins and integral spins). Fundamental interactions occur only between matter and interactions fields." For two particles, ψ = |1,1⟩, an interchange of their momentum labels leaves the state unchanged for bosons but negated for fermions, $|k_2,k_1\rangle = -|k_1,k_2\rangle$. Bosonic Fock space allows any number of particles in an "i-th" state slot. But for fermionic Fock space, each state can either be occupied by one or by no particle due to the Pauli exclusion principle.

For fermions, $\hat{a}^\dagger$ and particle annihilation operator "â " also apply to Dirac 4-spinors {"bi-spinors"} and as well as plane waves with momentum k {$e^{ip.x}$ = exp(i[**k**·**x**- ωt]) } .



quantum mechanical harmonic oscillator ...
physics.stackexchange.com

LHO. Figure One.

Although QFT stresses operator valued quantum fields, there are alternatives. QM and "all types of QFTs can be formulated using path integrals" {formerly called Feynman's Sum over Histories}. "This approach avoids the formalism of operators on Hilbert spaces." Other approaches, Schwinger's for example, "do not use creation and destruction operators, but source fields instead. It is strictly a matter of taste, and, sometimes, expediency."

QFT is also called "**local** quantum field theory" – often meaning that an event or operation or a measurement can never impact or change the probabilities of events that are spacelike-separated ("no faster than light physics"). But long-range "entanglement" in QM suggests non-local enforcement of correlations between particles. This apparent puzzle is supposedly resolved by saying: "The locality of a QFT refers to the operator algebra. The (non-) locality of Bell's theorem refers to the states (rays) of the Hilbert space. These are <u>different</u> notions of locality, and they coexist peacefully." [Stack].

QFT fields can be made to look more like QM wave-functions. "Just as a quantum particle is described by a wave function that maps positions to probabilities (or rather probability amplitudes) for the particle to be measured at x, quantum fields can be understood in terms of wave <u>functionals</u>: ψ[ϕ(x)] that <u>map functions to numbers</u>, namely classical field configurations ϕ(x) to probability amplitudes, where $|\psi[\phi(x)]|^2$ can be interpreted as the probability for a given quantum field system to be found in configuration ϕ(x) when measured [Teller] .

<u>The "Square Root of Reality Theme," Factorization:</u>

<u>Factoring Reality (what to multiply together to get real numbers)</u>

*Def: Factoring (called "Factorising**"** in the UK) is the process of finding the factors: what to multiply together to get an expression or number.*

Classical physics is described largely by the mathematics of real numbers. Quantum theory functions at a deeper level and makes use of kinds of *"Clifford algebras"* using basis vectors that are "like" the square-roots of negative or positive real "numbers" *{or unit n×n matrices, I_n}* and include complex and "hypercomplex" numbers such as Hamilton's quaternions with three imaginary numbers {i ,j, k}, Pauli matrices, and Dirac "gamma matrices" as examples. These are processed at the quantum level, and then "squaring" brings them to our level of reality. Here are a few examples:

ψ(x,t): In ordinary NR-QM, the wavefunction ψ(x,t) amplitude has to be "squared" to provide measurable probabilities of physical outcome: prob = $\psi^*\psi = |\psi|^2$. Intuitively, we could say that the wave function, ψ, by itself lives in what might be called *"the square-root of reality."* This might suggest that two-part back-and-forth "hand-shaking" agreement between field sources and absorbers enable measured probabilities *{for example, the "transactional interpretation of QM"}.* This sort of "factorization" theme is somewhat common and is also relevant to other basic concepts in quantum theory:

Cross-Section: This idea also applies to QED. Feynman Diagram Rules say that the overall amplitude is the coherent sum of the individual amplitudes for each diagram. And then the probability of scattering for two particles from initial to final states $\{ i \rightarrow f \}$ is given by the differential cross section dσ which in turn is proportional to the square of the invariant matrix element $|\mathcal{M}_{fi}|^2$.

E&M: For the special case of electromagnetic waves, if $B = \hat{k} \times E/c$ then E = cB. For the "Riemann-Silberstein" vector wave form **F**(x,t) = **E**(x,t) + ic**B**(x,t), $F^*F = E^2 + c^2B^2 \sim 2u_E/\epsilon_o + 2\mu_o u_B c^2 = (2/\epsilon_o)(u_E+u_B)$ where $\mu_o\epsilon_o = 1/c^2$ and u is field energy density. Wave forms F* and F are factors of energy density. Electric field E by itself is an "amplitude" that needs to be squared to get electric energy density.

Dirac: The relativistic "Dirac equation" effectively uses the "square root" of the d'Alembertian "□" = $\partial_\mu \partial^\mu = (c^2 \partial^2/\partial t^2 - \nabla^2)$ of the "Klein-Gordon" equation:

$(\Box + m^2)\phi = 0$ "→√→" $\mathbf{(i\,\gamma^\mu \partial_\mu - m)\psi = 0}$ based on 4×4 Dirac "gamma matrices" obeying γγ = ± 1 as operators. {That is, $(\gamma^k)^2 = -I_4$ and $(\gamma^o)^2 = (\gamma^5)^2 = I_4$. Or, equivalently, Dirac originally wrote his equation using 4×4 "alpha" matrices $\alpha_o, \alpha_1, \alpha_2$ and α_3 whose squares gave + I_4 and where $\gamma^i = \alpha^o\alpha^i$}. Gamma matrices are composed of 2×2 complex Pauli matrices, σ^1, σ^2, and σ^3.

Dirac wasn't aware that Clifford and Hamilton had similar discoveries to his in the 1800's. Algebras with a number count of bases, p, that square to +1 and q bases squaring to -1 are called "Clifford algebras" over real or complex numbers $C\ell_{p,q}(C)$. Those having p + q = even *{2n}*, are matrix algebras which have a complex representation of dimension 2^n. *{Real numbers are $R \simeq C\ell_{o,o}$, complex numbers are $C\ell_{o,1}$, Hamilton quaternions are $H=C\ell_{o,2}$}.* The Dirac Algebra $C\ell_{3,1}(R)$ or $C\ell_{1,3}$ has dimension 16 [Wik]. All of these matrices can be considered as operators on "spinors" ψ s.

This factor space is much broader than Minkowski $\Re^4$ and allow for new physical effects such as "antimatter" which are not found in classical space {for example, Charge conjugation could be $\hat{C} \sim i\gamma^2\gamma^o$, Parity operator $\hat{P} \sim e^{i\phi}\gamma^o$, Time reversal $\hat{T} \sim i\gamma^1\gamma^3$ so that $CPT = i\gamma^o\gamma^5$ where $\gamma^5 = i\gamma^o\gamma^1\gamma^2\gamma^3$}.

The "octonions" are a normed division algebra; but they are not a Clifford algebra, since they are nonassociative.

"Spinors:" It is sometimes said that fermion spinor "column vectors" are somewhat like "the square root of a vector" – but really more convoluted. One statement is, spinors are "essentially a two-component vector-like quantity in which rotations and relativistic Lorentz boosts are **built into** the overall formalism [Straub]." Their description during rotations defines them as objects that rotate twice around to regain their original state. And that is like "square-rooting" *{√ phasor ≃ ($e^{i\theta}$) ½ = $e^{i\theta/2}$ so that $e^{i4\pi/2}= e^{i(2\pi)} = 1$ making use of rotation "half-angles" for fermions }.* So, Penrose says that a spinor is an object which turns into its negative when it undergoes a complete rotation through 2π .

The great mathematician Atiyih said, "No one fully understands spinors. Their algebra is formally understood, but their general significance is mysterious. In some sense they describe the "square root" of geometry and, just as understanding the square root of −1 took centuries, the same might be true of spinors."

Creation operators: And another relevant example is that when studying the "creation operators" for raising the discrete energy level of the linear harmonic oscillator (LHO), one could notice that the quantum field 'creation' *(or 'raising,' 'exciting' operators 'a-dagger" = $\hat{a}^{\dagger}$)* and annihilation operators *(lowering or de-exciting $\hat{a}$*) are formed by the "square-root theme": Energy for the LHO = an offset T+V -- or rather 'number of rungs on the oscillator energy "ladder." $E_n/\hbar\omega$ **factorizes** into a and $a^{\dagger}$ {"a-dagger" or a* as sum and differences of x and p's} so that quanta number **n = a*a product of factors**).

These a's have to be promoted to operators with "commutators" in order to duplicate the results of the usual Schrodinger LHO ground state and also to actually perform raising and lowering using commutators. The Hamiltonian for the simple LHO is $H = \hbar\omega(\hat{a}^{\dagger}\hat{a} + ½)$. Zee said, "the whole subject of quantum field theory remains rooted in this **harmonic paradigm"** [Zee].

Quantum Electrodynamics, QED:

"In particle physics, quantum electrodynamics (QED) is the relativistic quantum field theory of electrodynamics usually as a perturbation theory of the electromagnetic quantum vacuum." "It describes how light and matter interact and is the first theory where full agreement between quantum mechanics and special relativity is achieved [WIK]." The fermion operator equation Eqn. 6 below is an example of how the creation and annihilation operators can be used. Only a glimpse of key math is shown here.

The relativistic electromagnetic tensor $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ is antisymmetrical so that its diagonal is zero; and the electric field is $E_i = cF_{oi}$ *(SI units with metric signature = (+ - - -)).* For classical Lagrangian density, $\mathcal{L}$, we have Action =

$$S = \int \mathcal{L}\, dx^4 = \int (-1/4\pi)\, (\mathbf{F_{\mu\nu} F^{\mu\nu} - J^{\mu} A_{\mu}})\, dx^4 , \text{ and } F_{\mu\nu} F^{\mu\nu} = 2(B^2 - E^2/c^2) \qquad \textbf{Eqn. 1.}$$

*with 4-vectors for current J = (ρc,J), position X = (ct,x), and vector potential $A = A^{\mu} = (\phi/c, \mathbf{A})$. The **4-gradient** covariant components, $\partial_\mu = (\partial_t/c, \nabla)$ state partial differentiation with respect to 4-position X^{μ} , and all signs are positive. Its contravariant components are $\partial^\alpha = \eta^{\alpha\beta}\partial_\beta$ has mixed signs. {if we switched to a metric (- + + +), then $F_{\mu\nu} F^{\mu\nu} = 2(E^2-B^2/c^2)$ as an invariant, and $E_i = -cF_{oi}$}.*

"The total energy of the multimode radiation field is given by {SI units}:
$H = ½ \int (\epsilon_o E^2+\mu_o H^2)dvol = ½ \int (D\cdot E + B\cdot H)\, dv$, where $B = \mu_o H$ and $D = \epsilon_o E$.

The non-interacting field Lagrangian becomes $F_{\mu\nu} F^{\mu\nu} = 2(B^2-E^2/c^2) = 0$!

The classical wave equation for the vector potential field, A, □A = 0 , may be expressed in a "standard separation of variables" for space-time, r and t:

$$\vec{A}(r,t) = \sum_m \sqrt{\frac{hbar}{2\omega_m \epsilon_o}} \; [a_m(t)u_m(r) + a_m^\dagger(t)u_m^*(r)], \qquad a_m(t) = \exp\,(-i\omega_m t)$$

{Eqn. 2.} where $\mathbf{u}_m(r) = \mathbf{e}_m \exp(ik_m \cdot r)/\sqrt{\ }$vol . The dagger here is just complex conjugation in anticipation of its later use for creating particles in quantum field theory.

Equation 1 expresses Lagrangian mechanics for <u>fields</u> or field energy rather than direct kinetic and potential energy terms. But the fields φ = (ϕ,A) are classical fields.

A "classical field" is a physical quantity that is "defined at every point in space-time," $\phi_a(x, t)$. They can include the usual examples of 3-vector Electric field E(x,t) and Magnetic field B(x,t) which in turn are derived from the 4-component field $A^\mu(x,t) = (\phi, A)$ in space-time.

Maxwell's equations follow from $F_{\mu\nu}$ and least action for S.
Minimizing an S = ∫ Ldt {-- having stationary action} leads to equations of motion or "**Euler-Lagrange equations"**]. For generalized coordinates q_i, these are

$$\frac{dL}{dq_i} - \frac{d}{dt}\left(\frac{dL}{d\dot{q}_i}\right) = 0 \;\; \rightarrow \;\; \frac{\partial L}{\partial \varphi} - \partial_\mu \left(\frac{\partial L}{\partial\,[\partial_\mu \varphi]}\right) = 0 \qquad \text{Equation 3.}$$

where the q_i's become fields φ, and over-dots become 4-gradients. That is, in a **<u>covariant formulation,</u>** time is placed on equal footing with space, so the coordinate time as measured in some frame is part of the configuration space alongside the spatial coordinates (and other generalized coordinates). The Euler-Lagrange {EL} result for equation 1 above is the combined "Gauss-Ampere" law: $\partial_\mu F^{\mu\nu} = \mu_o j^\nu$.
{And remember that the EL equation for Newtonian L=KE-PE just results in Newton's law, F = ma}.

For Quantum Electrodynamical [QED] in Dirac Theory, the Lagrangian density is developed from equation 1 above into:

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma_\mu \partial_\mu - m)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma^\mu\psi A_\mu, \qquad \bar{\psi} = \psi^\dagger\gamma^o \qquad \text{Eqn. 4.}$$

The coupling of electron and A-field is thus more than just eA.

"Quantum optics describes light using the theory" of QED. Quantum states are "driven by the Hamiltonian

$$\hat{H} = \tfrac{1}{2}\int \left(\hat{\vec{E}}^2 + \hat{\vec{B}}^2\right) d\vec{r} \;\simeq\; \textstyle\sum_k \hbar\omega_k \left(\hat{a}_k^\dagger\,\hat{a}_k + \tfrac{1}{2}\right) \qquad \text{Eqn 5.}$$

where the field vectors can be derived in the usual way from a vector potential, A. Here, E, B, and A are all quantum operators in a sense like the creation and annihilation operators [Search]].

When energy is delivered to an interaction event, a great many outcome possibilities may be stimulated from the 'Vacuum.' "The interacting ground state |Ω ⟩

becomes different from the non-interacting ground state |0⟩, and the dispersion of the particles is also altered *{ω = ω(k) or E = E(p) }* [Lancaster, Chap.31]. "If we put a test particle into this system it will interact with particles and antiparticles pulling them out of the vacuum" {pair-production}. Single particle excitations can still occur, but the particles become "dressed particles or quasiparticles." Effective masses can be different from their free mass values. The created state $a^{+}_{p}|\Omega\rangle$ can consist of "several excitations all at once" – "multiparticle states whose momenta each add up to **p**."

If an outgoing particle is a resonance with a decay time, it can have a complex energy $E_p+i\Gamma_p$.

The "Dirac Spinor field" can be decomposed into its Fourier moments [Kaku p 86] – fermion field operator:

$$\hat{\psi}\,(x) = \sqrt{\frac{m}{k_o}}\,\frac{d^3k}{(2\pi)^{3/2}}\sum_{\alpha=1,2}\left[\hat{b}_\alpha(k)u_\alpha(k)e^{-ik\cdot x} + \hat{d}^\dagger_\alpha(k)v_\alpha(k)e^{ik\cdot x}\right] \qquad \text{Eqn. 6}$$

Where "u" is the electron 4-spinor and "v" refers to a positron 4-spinor (electron going backwards in time). The upper two slots of the u column vector state spin-up or spin-down, while the lower two slots indicate the contribution of antimatter which increases with the energy/momentum of the particle. For psi as an operator, it is the b operator annihilating positive energy electrons and the d-dagger operator creating a positive energy positron. For fermions, the b and d operators must obey separate fermion anticommutation relations.

The spinors themselves are just columns of numbers (the "spinor" field doesn't have spinors as operators).

{In QFT, treating a field as an operator, $\varphi \rightarrow \hat{\varphi}$, is sometimes called "2nd Quantization." The term "first quantization" in ordinary QM refers to promoting x and p to operators and then imposing the "commutation relation" $[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$ -- particles act like waves. The number occupation state $|2\ 1\rangle \propto (\hat{a}_1^\dagger)^2(\hat{a}_2^\dagger)\,|0\rangle$ is similar to turning waves into particles}.

Calculations are often performed using Feynman diagrams. They have the visual appearance of particle interactions in space-time. But, "according to the received view Feynman diagrams do not represent or model the underlying physical process in any closer meaning of these terms. Instead, these diagrams are a calculational tool **or "bookkeeping device"** for perturbative calculations in quantum field theory [Passon] Expressed pointedly, they visualize formulae and not physical processes.

This view is in apparent tension with scientific practice in high energy physics, which analyses its data in terms of "channels."

Feynman believed the graphs are more than bookkeeping . He regards the graphs as a **picture of an actual process** which is occurring physically in space-time." (unlike Dyson, 1951, p. 99) -- Differing Interpretations of Feynman Diagrams is called THE FEYNMAN-DYSON SPLIT.

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https://souravchatterjee.su.domains//qft-lectures-combined.pdf

25. Overflow Notes.doc : 30 pages of notes that were deleted from this paper as not key.

Other Philosophical Notes:

[**Auyang]**: How is Quantum Field Theory Possible: p. 19: An operator A is a linear transformation of the Hilbert Space H into itself. $\mathcal{H} \rightarrow \mathcal{H}$.
*P45 **Two types of fields are distinguished: matter fields and interaction fields** with quanta of fermions and bosons (half-integral spins and integral spins). Fundamental interactions occur only between matter and interactions fields.
P51 **Operators are transformations of the states of the field.**
Particles are the normal modes or quanta of excitation of the field.
52 the operator A_{k_i} de-excites a quantum in the momentum mode k_i
p53 All interacting field theories share a common structure; they are all field theories with local symmetries.
P 57 the coupling of e and A is - $\bar{\psi}(x)e\,\gamma^{\mu}(x)\psi(x)$); not just eA.
EXISTENCE:
P75 ``A characteristic has empirical ramification if it is either observable or **``kickable**.'' Kickability is Alfred Lande's term. Something is kickable if it can be kicked and kicks back… If I turn a knob on my radio, the music changes. Quantum phase is kickable via the "AB" effect of vector potential A. ["Aharonov-Bohm", Auyang]

78 ψ is supposed to be a summary of quantum properties, so specific eigenvalues cannot be properties of quantum objects. EVs are not kickable within QM
the relativistic vacuum of QFT has the even more striking feature that the expectation values for various quantities do not vanish, which prompts the question what it is that has these values or gives rise to them if the vacuum is taken to be the state with no particles present. If particles were the basic objects of QFT how can it be that there are physical phenomena even if nothing is there according to this very ontology?

[McM] p 4 in addition to quantum operator field φ, there are momentum fields π(x,t) such that commutator $[\varphi(x,t),\pi(y,t)] = i\hbar\delta(x-y)$ – expressing causality.

In quantum theory, fields have a lot of spontaneous activity. They fluctuate in intensity and direction. And while the average value of the electric field in a vacuum is zero, the average value of its square is not zero. That's significant because the energy density in an electric field is proportional to the field's square. The energy density value, in fact, is infinite.

The spontaneous activity of quantum fields goes by several different names: quantum fluctuations, virtual particles, or zero-point motion. There are subtle differences in the connotations of these expressions, but they all refer to the same phenomenon. Whatever you call it, the activity involves energy. Lots of energy — in fact, an infinite amount. https://www.quantamagazine.org/why-feynman-diagrams-are-so-important-20160705/
What they show are not rigid geometric trajectories, but more flexible, "topological" constructions, reflecting quantum uncertainty. In other words, you can be quite sloppy about the shape and configuration of the lines and squiggles, as long as you get the connections right. … So if the universe contains an artfully balanced mix of bosons and fermions, the infinities can cancel. Supersymmetric theories, which also have several other attractive features, achieve that cancellation.
Another thing we've learned is that in addition to fluctuating fields, the vacuum contains non-fluctuating fields, often called "**condensates**." One such condensate is the so-called **sigma condensate;** another is the Higgs condensate. WILCZEK

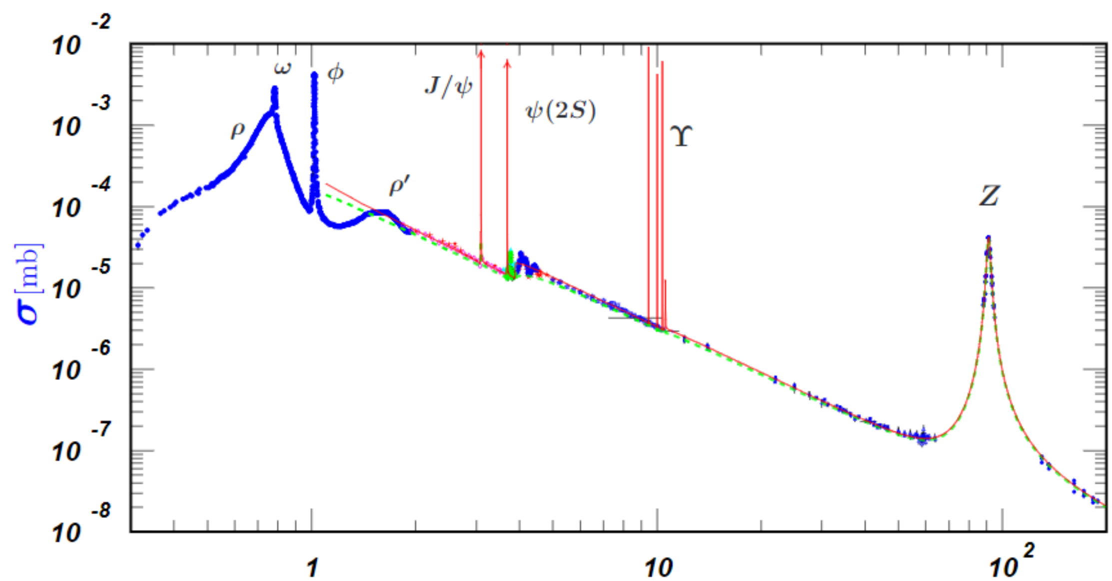
Addenda {About Quantum Field Theory}

10/6/22 – 10/26/22 *Dave Peterson*

Re. "About Quantum Field Theory," October, 2022 on Bill's cosmology net:
[About] *Dave*, 2022, http://www.sackett.net/AboutQuantumFieldTheory.pdf

Some Clarifications:

1. Remember, we are not doing the mathematics of field theory. We may encounter a lot of it; but we are only interested in the key sentences surrounding it that help interpret what the math is about and how the physics might be interpreted -- what could really be going on.
2. An experimental plot of actual particle creations of heavy bosons stimulated out of the Vacuum fields by electromagnetic photons (probability versus energy in GeV).
3. Don't fear the word "Lagrangians."
4. Creation of photons in our everyday world.
5. Some key sentences from readings.



(2) Figure 1: Particle creation out of the Vacuum, *from the "Particle Data Group":*

The cross sections **σ** for "quarkonia" (quark-antiquark meson resonances) production from high-energy electron-positron e^+e^- annihilation versus center-of-mass energy √ s in GeV. All of these particles are neutral spin-1 vector bosons.

Ref: https://pdg.lbl.gov/2022/reviews/contents_sports.html .

Through increasing collision energy to the right on the x-axis, neutral ρ and ω mesons are created first for u-and-d type quarks ($u\bar{u}$ -$d\bar{d}$), ($u\bar{u}$ +$d\bar{d}$). Then the ϕ-meson from strange quarks $s\bar{s}$; the J/ψ from charm $c\bar{c}$; the upsilon ϒ from bottom b-quarks $b\bar{b}$; and finally the neutral $\mathbf{Z^o}$ electroweak boson or "heavy photon." ψ(2s) is an excited state of the J/ψ (1s) $c\bar{c}$. This same plot would also result from **muon** $\mu^+\mu^-$ collisions *[as 'deduced' from LHC-CMS DiMuon output 2011 at √ s = 7 TeV].* The colliding leptons annihilate into photons which then excite the resonances out of the vacuum.

This is an example of the sentence, "When energy is delivered to an interaction event, **a "bubbling cauldron" of outcome possibilities are** evoked from the 'Vacuum' [Lancaster]. In this case, quarkonia pairs are examined.

(3): Don't fear Lagrangians {For us, Just a Machine to get "Action"}:

Most of the texts and articles on QFT include "Lagrangians" *(but don't stop reading when you see that word). We only care about its physical interpretation rather than details of its math.*

In classical mechanics, an expression called a Lagrangian, L, is written down in terms of energies and the use of appropriately clever new general coordinates applied to a chosen trajectory path in phase space ($x(t)$, $\dot{x}$). This "Lagrangian mechanics" is really just a different formulation of **Newton's laws** expressed without using forces or vectors *(energies are scalars)*. The output from processing a Lagrangian is a somewhat strange thing called "action," $S = \int L dt$. This is in turn then processed by a "least action" postulate which cleverly gives "equations of motion" that can then be solved for a given problem system. This "least action" is accomplished using "Euler-Lagrange" equations, an **"EL"** machine *{e.g., making $\partial L/\partial x - (d/dt)(\partial L/\partial \dot{x}) = 0$}*.

So we have the process:

[{a path and special coordinates} → **L** → **S**(action). And, **L** → **EL** → "equations of motion." **]**

In classical mechanics, **L ≡ KE-PE** (kinetic energy minus potential energy) *sometimes "L=T-V".*

Math: Also of great importance is total energy "Hamiltonian" defined by $H \equiv p\dot{x} - L = m\dot{x}^2 - ½ m\dot{x}^2 - - V = ½ m\dot{x}^2 + V = KE + PE = E_{total}$. Formally, the momentum "p" also comes from L, $p = dL/d\dot{x} = d(½ m\dot{x}^2)/d\dot{x} = m\dot{x} = mv$. The Schrodinger equation is nothing more than the E = KE+PE Energy in operator form: $(\hat{H} = \hat{p}^2/2m + \hat{V} = \hat{E})\psi(x,t)$. All of this can be generalized to fields, $\varphi(x)$.

Process Examples:

(a) For linear motion with KE = $½ m\dot{x}^2$, KE → [$\mathcal{EL}$] → **"F=ma"** or $m\ddot{r} = -\nabla V(r)$ -- which is just Newton's Law (derived).

(b) For the {very important} simple harmonic oscillator, L = KE – PE = ($½ m\dot{x}^2 - ½ kx^2$) →[$\mathcal{EL}$] → "$m\ddot{x} = -kx$." *{(force is inversely proportional to the stretching of a spring with spring constant k, and the solution has a natural frequency $\omega_o = \sqrt{(k/m)}$}.*

(c) Harder: *{For the special "Klein-Gordon" Lagrangian (of a spin zero relativistic field), the EL machine yields the Klein-Gordon equation, $(\square + m^2)\varphi(x,t) = 0$ – to be solved for φ – often in terms of plane-waves }.*

It turns out that this process is broader and more useful than just Newton's equations and also applies to relativity and quantum fields and electromagnetism as well.

"Action" is found by summing up the values of L for all points along a trajectory, $S = \int L dt$. This concept of "**action" is a bit opaque** until we come to quantum mechanics, QM. Then there are clear physical ideas that make it all work out intuitively – action is accumulated phase of "waves," and "least action" is an end point that makes accumulated cycles smallest. That is, in ordinary quantum mechanics, the Lagrangian, L, is just an expression or "machine" that enables the counting of wavelengths (phase) along a path, and that is proportional to "Action," $S = \int L dt$.

{recall that wave "phase" ϕ (in radians) = $2\pi(\nu t- x/\lambda) = \omega t-\mathbf{k}\cdot\mathbf{x} = k_\mu X^\mu$ – how far you've gone through a stream of wave cycles. Our goal is looking for constructive interferences of paths } .

The common "L = KE-PE" energies only involve velocity and potential versus position but not usually time as variables. "Phase" $\Delta\phi \simeq L\Delta t/h = S/h$; or the number of wavelengths or cycles $n = L\Delta t/\hbar$ [Ref. "Learn"]. Planck's constant "h" itself is also a tiny bit of action in units of joules per hertz = J·sec *{so that $E = \hbar\omega = h\nu$ }.* And so also are Bohr's circular orbits. So, say, an $n_{Bohr} = 3$ orbit with 3 waves around a circle would have 3 h's of action.

In physics, the word "symmetry" usually means "the symmetry **of** the Lagrangian." *{popular books don't write this because the word "Lagrangian" might scare readers}.* So, L not being a function of time means energy is conserved and not being a function of position would mean momentum would be conserved.

One interesting point is that the "frequency" in wave mechanics deletes the contribution of "mass energy;" while $\omega = E/\hbar$ in relativistic theory includes mass energy ($E = \gamma mc^2 = m_o c^2$+KE -- thus giving very high frequencies > 10^{20} Hz). And the Lagrangian changes to $L = -m_o c^2/\gamma$ – PE but still represents wave-counting. In QFT equations, we have both particles and waves in the same terms – (particles created) times the wave $\exp(i\,[\mathbf{k}\cdot\mathbf{r}])$ -- e.g., eqn. 6 in *[About]* .

QFT changes from wave functions ψ in QM to "operator fields" but still seems to have a wave-counting property to its Lagrangian. A side mystery is then how classical mechanics works. Its Lagrangian is often L = KE-PE, but no waves seem to present. Newton's laws are treated as a given—but they should be derivable from quantum mechanics {?}. *{The Ehrenfest theorem of 1927 is not adequate}.*

For QFT "operator fields," note that formulas using creation operators have to include annihilation operators too (added or multiplied because each by itself alone isn't kosher *{called "Hermitian"}*). Some examples are $\hat{H} = \hbar\omega\,(\hat{a}^\dagger\hat{a} + ½)$ is Hermitian as is $x \propto \hat{a} + \hat{a}^\dagger$ and $p \propto \hat{a} - \hat{a}^\dagger$.

(4): *An innocent question, "What is the simplest example of particle creation in the realm of our daily lives?"*

Light from Spontaneous emissions (creation of photons):

High energy particle physics and its quantum field theories are far removed from our everyday experiences. More highly relevant is the world of light that we see all around us. And its photons are mostly due to spontaneous emission *{SE, or "radiative decay" or "luminescence" }* from excited states of atoms and molecules to their lower energy states. Since photons are being created and absorbed, their proper description requires quantum field theory *{e.g., QED}.*

SE in free space is generally understood as depending on vacuum fluctuations "to get started." That is, the ground state of the quantum harmonic oscillator is not zero energy but effectively the energy of a "half-photon," ½ ℏω, that is also called "zero-point energy." The frequency can take any value. The Heisenberg uncertainty fluctuations of this base state perturb surrounding fields and can stimulate decay *(but, it has been unclear if these scenarios make completely coherent sense [Sybil]. Despite general belief, the big issue is that if zero-point energies are there and have immense contribution, why don't we see them in the energy of the universe?).*

As an example of decay, in the transition of an electron in a free hydrogen 2p atomic state to 1s state, a UV photon of λ ~ 121 nm is emitted on average in about 1.6 nanoseconds. But, the 2p is called a stationary state and shouldn't decay unless there is some hidden time dependence [Torre]. However, the QED "interaction between the electron and photons renders the original non-interacting stationary states no longer stationary." A photon transition involves the mixing of the 2p and 1s states resulting in an "electric dipole moment" vibration *{e.g.,* $d_{nm} = \langle \psi_m | q\mathbf{r} | \psi_n \rangle$ *so* $d_{21} \sim 0.75(ea_o)$ *}* in a beat frequency of the difference between the 2p and 1s frequency for the released photon.

Spontaneous emission decay is <u>not</u> just a property of an isolated atom but also depends on its association with the quantized electromagnetic field, vacuum noise, and the modes that are available. Emission can be totally suppressed inside a cavity where dimensions will not support vacuum standing waves of the needed transition wavelength *(Purcell effect).* The cavity wall modifies, enhances, or limits the vacuum fluctuations background field that can couple to an atom; an atom in an excited state is not quite a stationary state like H(2p) because the atom couples to the quantum electromagnetic vacuum. Then, one could speak of SE as emission "**stimulated** by a vacuum photon."

But, despite common acceptance, most cases of physical phenomena believed to involve vacuum fluctuations can also be accounted for without them. "Instead, the field radiated by the charge itself — usually called either the "radiation reaction field" or the "source field" can explain results. This is true not only for spontaneous emission but also for the famous Casimir Effect [Sybil]. So, the simplest example of photon decay may not really be so simple.

Another confusing point is a definition of <u>what a photon is.</u> It is said that a "photon" is the activation of an elementary excitation of a mode of the quantized field { $|1\rangle = \hat{a}^\dagger |0\rangle$, details [stack eγ] }. A usual picture of treating photons as "particles" during flight is problematic because there is no position operator, $\hat{X}$, in QED. We think of particles as capable of being localized; but photons cannot be localized except during creation and detection. So, a clear definition of <u>existence</u> for photons has been elusive. "The photon is an <u>**event**</u>, not a thing." Annihilation of a photon takes energy away from a photon and gives it to a detector.

5. <u>Some Key Sentences from the Readings {References from [About] }.</u>

[Wallace-D]: "we can see that Lagrangian QFT (as I have defended it) is not really in conflict with AQFT at all. Success in the AQFT program would leave us with a field theory exactly defined on all scales, and such a theory would be a perfectly valid choice for 'theory X': furthermore, even if we found such an exact QFT it would not prevent us from defining low-energy, 'effective' QFTs — which would not be well defined without a cutoff; "

[Wallace-E] Re: condensed matter physics: "There are striking formal parallels with quantum field theory: in fact, the construction of phonons from a monatomic crystal is virtually the same as the construction of particle states in a massless, scalar quantum field theory. The difference is, the ontology of a crystal is not in question."

"For in a generic solid-state system, atoms are coupled to their neighbours, and as a consequence the ground state of the system is highly entangled. This allows us (in principle) to exploit the long-range correlations between spatially separated subsystems of the field…"

"it is satisfying to find that field-particle duality can be understood in the context of a field ontology,…"

Quote: "I am not sure it is necessary to formulate the foundations of QFT, or even to precisely define what it is. **QFT is what quantum field theorists do.** For a practising high energy physicist, nature is a surer guide as to what quantum field theory is as well to what might supersede it, than is the consistency of its axioms". (David Gross, 1999, p. 56).

Multiphoton **entanglement** superpositions are just "a way to enforce conservation laws given a world of possibilities." So the real mystery is **how conservation laws are enforced.**

NOTE: **on [about] Ref. 4**, **Fraser** -- no immediate web connection – Mike says: "About reference 4 in the original: All that is necessary is to remove the trailing "https:/" at the end of the url. Then the link works." How to take particle physics seriously: A further defence of axiomatic quantum field theory - ScienceDirect https://www.sciencedirect.com/science/article/abs/pii/S1355219811000141

New References:

[Sybil] "Fluctuations of the Electromagnetic Vacuum Field or radiation reaction?" http://jamesowenweatherall.com/wp-content/uploads/2014/10/Sybil_de_Clark_April11.pdf

[Torre] C. G. Torre **What is a Photon?** Foundations of Quantum Field Theory June 16, 2018 107 pgs https://digitalcommons.usu.edu/physics_facpub/2066/ also see: (b)
http://www.physics.usu.edu/torre/3700_Spring_2015/What_is_a_photon.pdf
https://digitalcommons.usu.edu/cgi/viewcontent.cgi?article=3211&context=physics_facpu

[Learn] Dave, http://www.sackett.net/DP_Stroll2.pdf "Learning Quantum Mechanics and Relativity," wave-counts on pg 42 and 47/381 in Stroll II. & "Photons" and Light, dp, 2019 on pgs. 75-89, and a list of 11 "Problems needing answers" p 29/381.

[stack-eγ] https://physics.stackexchange.com/questions/95690/how-do-electrons-and-photons-interact/95702#95702, $\hat{H} = \hat{H}o + e\hat{r} \cdot (E_o\hat{a} + E^*_o\hat{a}^\dagger) + \hbar\omega(\hat{a}^\dagger \hat{a} + ½)$.

https://www.informationphilosopher.com/books/problems/Entanglement.pdf free books.
https://profoundphysics.com/lagrangian-vs-newtonian-mechanics-the-key-differences/