Concrete Hidden Variables __ Rev. 4

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Overview: Bell inequalities and tests for EPR experiments reveal that no local hidden variable can explain the observed correlations between two well-separated EPR detectors. Rather than inequalities, this study focuses more on continuous function plots of quantum mechanical (QM) results versus hidden variable correlations as a function of polarizer difference angles. Desiring an intuitive model that might approximate QM, we may suppose it to be due to predetermined polarizations emitted from a common source (pre-established coordination). And, without familiarity with concrete examples of what a hidden variable λ could really be, we might weakly retain this wrong idea in the back of our minds. Indeed one hidden variable (HV) example included here is fairly close to the correlation for real quantum mechanics (case HV-A, see **Fig. A and Fig. E**). But actual QM can do better than any Bell HV metric.

Surprisingly, a recent survey on the beliefs of physicists [10] showed that a third still say that physical properties exist prior to and independent of measurement, and only a third say that local HV's are impossible. But a third also have general ignorance of Bell tests. The ignorance of Bell is partly due to its difficulty, abstraction and strangeness. QM calculates and tests say that a photon observation in one detector immediately "snaps" the other photon into allignment. Hidden variables says that a more contrived or classical mechanism may approximate a similar output going back to the state of the photons when they were emitted.

John Bell presented his revolutionary "Bell Inequality" for the Einstein-Podolsky-Rosen (EPR) entanglement paradox in 1964. Using essentially classical and logical arguments, he showed that "any physical theory that assumes local realism cannot also predict all of the results of quantum mechanics [1]." He did this by introducing abstract local hidden variables (LHV's) represented by the symbol "lambda" and derived special Bell-inequalities that would be decisively violated by actual experiments for entangled EPR particles. His tests involves performing one experiment with a test-pair of set angles (a,b) and then another with a different set angle, c, and then comparing them. The first Bell statistical coincidence test looked something like this:

$$Correlation \quad C(a,c) - C(b,a) \; - \; C(b,c) \; \leq 1$$

Examples shown below include: (1) A derivation of the standard quantum mechanical correlation $P(V_a V_b) = \frac{1}{2} cos^2(b-a)$. (2) A derivation of the LHV "Triangle Plot" in **HV-A** and in Fig. A below. In figure C, this is shown as "overlapping bricks." (3) An interesting but puzzling non-local hidden variable beginning with λ's but ending up as quantum mechanical in **HV-B**. (4) A more intuitive LHV "colliding hills" setup in section **HV-C** (and not as good as HV A), (5) "**CHSH**_Bell" theory for quantum mechanics vesus hidden variables (figures D, and E for E(a,b)), and (6) some discussion of non-locality in QM. **All graphs of figures A and E are derived here.**

An example of one of Bell's early arguments is available in a recent paper for our Boulder Cosmology group [1]. Currently there are now many different types of test inequalities (e.g., "CSCH"), but they are all still called Bell inequalities. A great many well-tested and precise experimental violations of Bell inequalities show that all local hidden variable

approaches expressed by the symbol λ are doomed as a class. This was a major advantage of having a general abstract derivation. But, to really understand it intuitively, we need to show and picture some concrete possible examples of what a hidden variable might be.This paper largely avoids Bell inequalities and instead focuses on continuous graphs (like Figure A) for QM versus local hidden variable mathematics over all possible difference angles. The goal is to derive the plots showing in **figures A and E**.

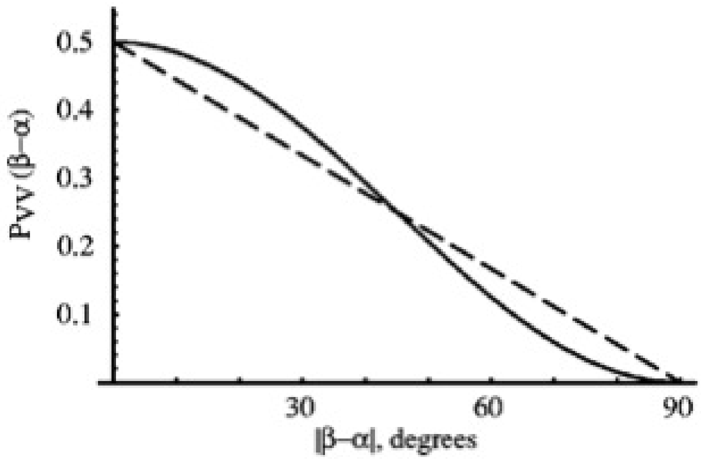
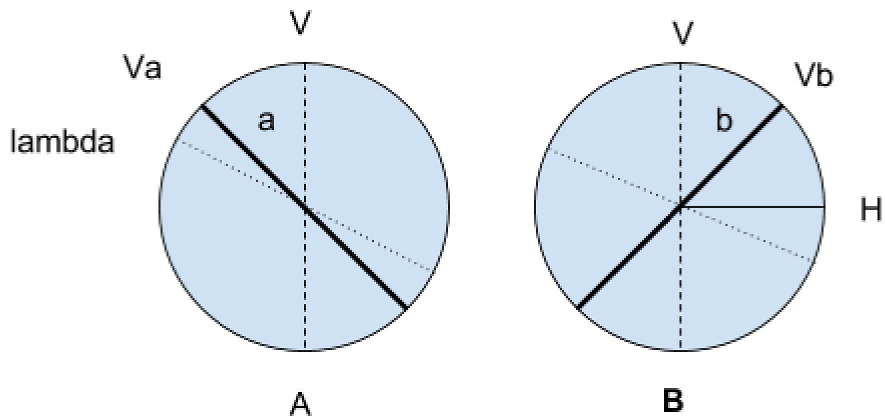


Fig. 4. Predicted polarization correlations for a quantum mechanical entangled state (solid curve) and a hidden-variable theory (dashed line).

Figure A: A standard plot [Ref. **2**] of the cosine curve for QM (solid) against the best hidden variable estimate (HV, dashed line) showing greatest relative difference angles of polarization near 20 or 70 degrees. Also look at a similar plot for estimations E(a,b) in Figure **E** below..

The initially proposed theoretical setup considered two entangled particle spin polarizations from a central singlet state having total angular momentum zero. Particles are directed to two different spacelike separated detectors labeled A and B for spin measurement orientations labeled a and b. For spin, this might be Stern-Gerlach magnets with different north-to-south rotation angles (a and b). But, is hard to actually do these spin angular momentum experiments; and it was found that use of photon polarizations was much more practical. To date, there has been no successful test of Bell's theorem using particle physics. However, a Bell test in 2015 [16] verified electron spin entanglement for electrons held in two stationary diamond nitrogen vacancy centers but still using photons propagating between them over a distance of 1.3 kilometers. This is not quite what Bell suggested, but it is still interesting.

Almost all experimental tests to date have been done using photons with an electric polarization basis that can be labeled as horizontal and vertical, H and V [see Fig. **B** and the upper portion of Fig. **D**], and considering just two entangled photons and two polarization detectors for the early experiments. For a frequent convenience here (which is allowed) we could set detector (say A to the left) to untilted xy polarizer axes (angle a = 0 = vertical V). Bell's key new idea was to consider rotating the other detector by an angle that is **not** the traditional b= 0, 45, 90, 135 or 180 degrees but rather some angle in-between--like 22.5 degrees (e.g., see graph differences in Fig. A).



Polarizer Tilt Angles for photon detection.

Figure B.

Early experiments used two photon decays from atoms in an atomic beam (e.g., excited Ca-40 decaying sequentially to green and violet entangled photons [6][14]). But now it much more convenient to get entangled photons from laser beams passing through nonlinear crystals (e.g., nonlinear crystal Type-1 down-conversion (DC) sources so that the two output photons both begin with the same polarization HH or VV or their superposition. A common example is a specially configured beta barium borate (BBO) type 1 crystal where a tiny fraction of the incident photons (about 10^{-10}) get photon entanglement by "spontaneous parametric down conversion" (SPDC). Laser photons passing straight through the crystal go into a beam dump and are ignored. To mathematically process this with ideal hidden variables, λ, for detectors A and B we need to specify: a name for the hidden variable, its probability distribution $\varrho(\lambda)$ [*which does not have to be uniform*] , explicit functions for individual measurements

$A(\lambda, a)$ *and* $B(\lambda, b)$ or <u>how</u> they are going to be used (what are the rules). We first require that

expectation values: $<A> \; = \; \int \varrho(\lambda) d\lambda \, A(\lambda, a)$ and $<B> \; = \; \int \varrho(\lambda) d\lambda \, B(\lambda, b)$.

And then for measured outputs using intuitive hidden variable, λ, and <u>joint</u> settings of a and b, we need to integrate over all lambda's [Ref. 1].

$$P(a,b) \; = \int \varrho(\lambda) A(a,\lambda) B(b,\lambda) d\lambda, \; where \;\; 0 \le \varrho(\lambda) \le 1 \;\; and \;\; \int \varrho(\lambda) d\lambda = \; 1. \, Eqn. \;\; 1$$

This is the "standard" Bell LHV form. Notice here that the LHV detector functions A and B only depend on their separate <u>local</u> settings a, b, and λ .For averages, we sweep through an ensemble of hidden variable values according to a probability density $\varrho(\lambda)$. The hidden variable here is: a pre-determined propagating input photon polarization angle $= \lambda$ [the very light lines in Fig. B] and possibly use of sine or cosine electric vector projections onto polarizer angles.

What makes the actual quantum mechanical (QM) case **<u>different</u>** from intuition is that polarization is <u>not</u> defined prior to measurement (passing through the polarizer to a detector). But as soon as one photon is detected the other instantaneously and non-locally "is projected into a state of polarization parallel to" the first result (V or H, see Ref [5], Aspect) . Whether this is a photon on either A or on B is totally random (collapse is random). I call this reduction "**SNAP-TO"** (as in a soldier snapping to attention or a computer visual "snap to grid"). This is different from the intuitive but naïve idea that perhaps the EPR photons were initially tilted at the same angle from the source and kept that alignment up to the time of detection (called "real"). For that case, the initial hidden variable polarizer angle could be anywhere from 0-180 degrees (i.e.,0 to π radians), a uniform distribution [$\varrho(\lambda) \; = cnst. = 1/\pi$ radians], so that

$\int \varrho(\lambda) d\lambda = \pi(1/\pi) \; = 1, \;\; \lambda \in [-\pi/2, +\pi/2] \; or \; [0 \; to \; \pi]$ -- an ensemble of all possible predetermined polarization angles. This provides a <u>concrete example</u> where the hidden variable lambda is merely any predetermined tilt angle for both of the photons per event.

<u>QM:</u> **<u>Actual Quantum Mechanics Calculation</u>** Let's begin by first looking at the actual physical **QM** calculation for the coincidence of detector hits for <u>vertical</u> polarizer angles [2]. Begin with a left polarizer **A** having angle a = V = "|" and right polarizer B having angle b and V_b "/" and look at probability coincidence P(VV) meaning V's being vertical <u>in the</u>

<u>tilt angle bases</u> of their respective polarizers [Fig. **B**]. Let the initial polarization state of two entangled photons be given in a neutral untilted basis: **<u>Eqn. 2</u>**

$$|\psi_{EPR}> \;=\; (1/\sqrt{2}\;[\,|V>|V> \;+\; |H>|H>]\;,\; so \quad |V_a> \;=\; cos\, a|V> \;-\; sin\, a|H> and$$

$$|H_a> \;=\; sin\, a\,|V> \;+\; cos\, a\,|H>.\;\; Let\, |\psi_{DC}> \;=\; cos\theta|H_1>|H_2> \;+\; sin\theta\,|V_1>|V_2>,$$

where "DC" means "down conversion," entangled photons are 1 and 2 or L and R on the untilted vertical (y axis) and horizontal (x axis). And we make the initial laser beam entering the nonlinear crystals for down conversion to have polarization oriented at $\theta = 45^o$ *so that* $cos\theta = sin\theta = 1/\sqrt{2}$. Then the "balanced" $|\psi>_{DC} = |\psi>_{EPR.}$ There are four basic types of 'Bell states", [aligned or un-aligned (e.g., VH) and superpositioned with a + or -], but we will only use this one $|\psi>_{DC}$.

Then project the down converted state onto "tilted-vertical" polarizations as in <u>Fig. B.</u> So,

$$P(V\,V) \Longrightarrow P(V_a V_b) \;=\; |\; <V_a|<V_b|\,|\psi_{DC}>|^2 = (\;1/\sqrt{2}\,)^2 \;\; |sin\, a\, sin\, b \;+ cos\, a\, cos\, b|\,)^2$$

{where trig coefficients are picked out like this: e.g.,

$<V_a|$ H> = <-sin a H|H> = -sin a <H|H>= - sin a },

$But \;\; cos(a-b) \;=\; cos\, a\, cos\, b + \; sin\, a\, sin\, b,\; so \qquad P(V\,V) \;= cos^2\,(a \;-\; b)\,/2$ ANS.

<u>Or,</u> if we set angle a at 0 and replace angle b by the difference angle $\alpha = \; b-a,$

$$P(V\,V) \Longrightarrow P(V_o V_\alpha) \;=\; |\; <V_o|<V_\alpha\;|\;\;|\psi_{DC}>|^2 = (\;1/\sqrt{2}\,)^2 \;\; |sin\,0\; sin\,\alpha \;+ cos\,0\; cos\,\alpha|\,)^2$$

$= \; cos^2\,\alpha\,/2$. (Again). We can rewrite this also as: [see Fig. **A**]

$$P(V\,V) \;=\; (1/2)\; cos^2\,(b-a) \;= (1/4)[1 + cos\; 2(b-a)]. \qquad Eqn.\;\; 3$$

The variable used for actual quantum mechanics tests and many LHV's is solely the <u>difference in tilt</u> angles of the two detectors [angle alpha = $\alpha = \;(b \;-\; a)$]. QM calculations result in a Bell correlation depending on cos(2α) [note: for fermion electrons it would be just -cos(α)]. Suppose again, by rotational symmetry and convenience, that the left device A-angle is vertical, a =0= "| "; and there is a lambda angle $\lambda \sim$ " \", and right detector tilt angle may be $b = \alpha$ = "/ ". For the quantum case, a hit on the vertical detector "|" <u>snaps</u> the other photon also to "|" so that the only relevant polarizer angle to project onto is alpha for the second detector, B. $\lambda\; \rightarrow \alpha$.

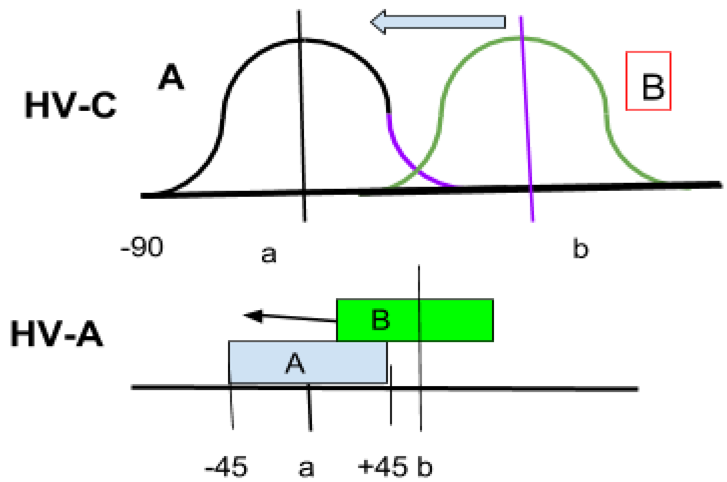


Figure C: Correlation Integrand shapes for overlap of A and B function shapes for Convolution. Cases "colliding hills" and "overlapping bricks."

Example HV-A: 2 joint snap-to's without Malus' law, "**The Triangle Plot"=** (Fig. **A**)

Now, what about those plots we've seen before of "naïve" **triangle** (capital Lambda shape or "tent map" e.g., Fig A) approximating the quantum cosine curve. Straight slanted lines go from the top of the QM cosine curve to its bottom valley – a "not too bad" approximation. Well, this case is for an implementation of hidden variable density using step-functions-- a special closeness and almost successful rule. This first concrete example is also the most popular example. Instead of vector electric projections to the polarization axes, here we "snap-to" when within an effective domain of angles. This special HV rule is that If angle $\lambda =$ " \" lies within 45^o of $\alpha =$ "/" and 0 = "|", then they both registers as V ="|" [e.g., ref. **2 see RULE**]. Here we are not using sine/cosine projections and not using the rules of quantum mechanics.

In electrical engineering (EE), the relevant $\varrho(\lambda)$ distribution is called a "rectangular" or Pi= Π shape with width $\pi/2$, $Fig.C$ [13]. The probability
$density\ \varrho(\lambda) = h\Pi(2\lambda/\pi) = height\ \ h = 2/\pi\ \ for\,|\lambda| \le \pi/4 = 45^o,\ (else\ 0).$
The total domain width of lambda is
$w = [\pi/2 - (-\pi/2)] = \pi,\ but\ the\ width\ of\ \Pi\ is\ \omega = \pi/2,\ so\ h = 2/\pi.\ (product\ unity)$

$$Then,\ \int_{-\pi/2}^{+\pi/2} \varrho(\lambda)d\lambda = \int_{-\pi/2}^{+\pi/2} h\Pi(2\lambda/\pi)\, d\lambda = 1.\ \ And,\ <A>= \int h\Pi(2\lambda/\pi)(1\Pi(2\lambda/\pi)d\lambda = 1,$$

(as required, if we center a at 0 -- and we can). As lambda moves from lower to upper of its range, the rectangle Π with λ as center moves with it.

The special EE shape functions $\Pi(x)\ and\ \Lambda(x)$ are both understood to have unit height, and the form $h\Pi((\lambda - a)/\omega)$ is a symmetric function centered at a, has total width ω and height h [13].

RULE: The rule for this LHV is "**We require λ to be close to both the left and right detector polarization** (A, B)"; and we care about the overlap correlation (angular overlap width = **W** < w radians, see overlapping W of Fig. C).
This is exactly the problem shown by animation in Wikipedia [3] for a convolution that outputs a Λ shape (please look, it is kinda neat!). By definition: a function f convolved with g is:

$$f(x) * g(x) \equiv \int f(u)g(x-u)du = \int g(u)f(x-u)du, \; let\; u = \lambda, \; here \int \Pi(0-\lambda)\Pi(\alpha-\lambda)d\lambda$$

Convolution calculations can be hard and often require numerical methods, but ordinary calculus can also be used [e.g., 7]. This case with flat-uniform distributions is much easier: there are no curves or slopes in these functions, so we can just apply simple geometrical thinking. Since we've conveniently chosen detector A to always have a vertical orientation (a = zero tilt from vertical), we can position the lambda density also at rotation zero leaving us with detector B with rotation alpha (the difference from b minus a angles, $\alpha = b - a$). Overlap enables both a and b to be in range of $\lambda = \pm\pi/4$. **Eqn. A0**

$$P_{VV}(\alpha) = \int \varrho(\lambda)\, A(0, \lambda = 0)\, B(\alpha, \lambda)d\lambda = (1/\pi)\int \Pi(2[\lambda - 0]/\pi)\, \Pi(2[\lambda - \alpha])\, d\lambda$$

This has the correct LHV form of Eqn. 1. And we also attain maximum probability when the difference angle $\alpha = 0$. (when full rectangles overlap)
$So\; P = h(W = w) = (1/\pi)(\pi/2) = 0.5$. There may be two domains to consider for overlap width W = right overlap minus left of overlap domain: [Slope Functions, **Eqn. A1**]

$$\alpha > 0 \Rightarrow W(\alpha) = \pi/2 - \alpha, \; and\; \alpha < 0 \Rightarrow W(\alpha) = \pi/2 + \alpha, \; so$$

$$P(V\,V) = (h)(1)W = [1/2 - \alpha/\pi].$$ Plot.

And these are just the equation for the triangle (Λ, Figure **A**) with left slope up and right slope down ending at $\pm\pi/2$ [Figure A]. For the fermion-electron case, the slopes would end at π -- twice as wide. Bell's theorem says that it is impossible to find any local hidden variable example that can give actual quantum mechanics results. So Example HV A modestly fails to agree with QM. But, this seemingly kludgy "double snap" ends up being much better than some other attempts with local hidden variables (such as HV C below).

Notice that the slope equations only pertain to P(VV) and shouldn't yet be compared yet to the more elaborate E(a,b) experimental estimate of the CHSH test.

Example HV-B [Possible Puzzling Exemption]: Simple Malus' Law projections of pre-existing photon polarization (angle λ onto two analyzer orientations a and b or 0 and

$\alpha = (b-a)$. This sounds like a hidden variable calculation and lambda might really project initially onto the polarizers. But then lambda gets discarded at the end. Overall, this is a little strange. Bohr might say that if you can't measure the effect of λ, then don't bother even thinking about it. But it could participate before it is subtracted away.

This time, a detection on the "|" detector leaves the other photon <u>where it was</u> at λ (no snap-to) so that the electric vector now has to project onto the <u>difference</u> in angles of $\alpha - \lambda =$ "/ - \". [We <u>are</u> assuming Malus' Law for real electric fields of a photon projecting onto the polarizer angle for detector A and separately for detector B]. Classical calculations may then require an integration over the ensemble of all local "hidden variable" λ angles.

<u>Derivation</u>: use of hidden lambda and trig projections onto two polarizer angles:

$$\varrho(\lambda) = (\,height\ h = 1/\pi)\ for\,|\,\lambda\,| \le \pi/2 = 90^o\ (else\ 0).\quad Then,\ \int_{-\pi/2}^{+\pi/2} \varrho(\lambda) d\lambda = 1.$$

$$P(V\,V) \Longrightarrow P(V_o V_\alpha) = \int_{-\pi/4}^{+\pi/4} |\, < V_o| < V_\alpha| \ \ (|V_\lambda > |V_\lambda > + |H_\lambda > |H_\lambda > |^2/2 \cdot \varrho(\lambda) d\lambda =$$

$$= ((1/2) \int [< V_o|V_\lambda > \cdot < V_\alpha|V_\lambda > \ + \ < V_o|H_\lambda > \cdot < V_\alpha|H_\lambda >]^2\ \varrho(\lambda) d\lambda =$$

$$= (1/2) \int (\ cos(0-\lambda)\ cos\,(\alpha-\lambda) + sin(0-\lambda)\ sin(\alpha-\lambda)\ )^2\ \ \varrho(\lambda) d\lambda \quad .$$ **<u>Eqn. B1</u>**

$$=(½) \int\ [sin^2\lambda\, sin^2(\alpha-\lambda)\ +\ cos^2\lambda\, cos^2(\alpha-\lambda)\ -2 sin\,\lambda\, sin\,(\alpha-\lambda)\, cos\,\lambda\, cos\,(\alpha-\ \lambda)]\ \ \varrho(\lambda) d\lambda$$

At first, this approach might be another Convolution Integral. **Goal**, for example,for each alpha, evaluate the integral for lambda and then sweep through possible alphas for a final plot of P(VV) versus the polarizer difference settings alpha.

e.g., A peak result occurs for alpha = 0: that is,

$$P(V\,V)(@\alpha = 0) = (0.5) \int (cos^2\lambda + sin^2\lambda)^2\ \ \varrho(\lambda) d\lambda = 0.5 \int 1\ \varrho(\lambda) d\lambda = 0.5\,(1) = 0.5 = 1/2.$$

e.g., And for $\alpha = 45^o$, $integrand\ A1 = cos\,\lambda((1/\sqrt{2})(cos\,\lambda + sin\lambda) - sin\lambda((1/\sqrt{2})(cos\lambda - sin\,\lambda) =$

$$= (1/\sqrt{2})(cos^2\lambda + cos\lambda sin\lambda) - (1/\sqrt{2})(sin\lambda cos\lambda - sin^2\lambda) = (1/\sqrt{2})(1),\ P(V\,V) = 1/4$$

<u>BUT</u>, look more carefully at that integrand in Eqn. B1. <u>Eqn. B2 :</u>

$$cos\,a\,cos\,b\ +\ sin\,a\,sin\,b\ \ = cos(a-b) = cos(0-\lambda)cos(\alpha-\lambda)\ = cos(0-\lambda-(\alpha-\lambda) = cos(\alpha)\ \ !!$$

And then: $P(V\,V) = P(V_o V_\alpha) = cos^2\alpha\,/2.$ <u>This result is just QM!!!</u> ANS.

The lambda contribution subtracts away! [I only noticed this after doing a spreadsheet calculation]. There is no need for convolution over all lambdas. The lambda angle can be anything or everything.

This hidden lambda pre-existing orientation ends up working like QM.

And, note that Eqn. B1 is **not in the LHV form of Eqn 1.** Contributions from A and B are mixed together (arguments with $(\alpha - \lambda)$ *and* $(0 - \lambda))$. But **Local** hidden variables require separating A and B in the HV equation.
Most HV equations begin with finding averages of results for tests A and B separately.

For example, $< B(v) > = \int \varrho(\lambda) d\lambda\, B(\lambda, v, \alpha)$ where v is a preparation direction (like H or V). If

we let $B(\lambda, \alpha) = |< V_\alpha || V_\lambda >|^2$, *then* $B = cos^2(\alpha - \lambda)/2 = cnst.$ (interesting) **Eqn B3.**

One might think that a proper approach should also include horizontals:

$B(\lambda, \alpha) = |< V_\alpha| (|V_\lambda > + | H_\lambda >)|^2 = |cos(\alpha - \lambda) + sin(\alpha - \lambda)|^2$

$= |\sqrt{2}\, sin(\alpha - \lambda + \pi/4)|^2$. And it is not clear that this should be dismissed. Overall, this example HV-B is puzzling and suggests extra thought.

HV-C: A more intuitive proper LOCAL hidden variable calculation using as input the cosine-squared **Hill**-profile of Figure **C --** somewhat like the QM Eqn 3 above and beginning with an optical Bell calculation for the individual detectors: Let

$$A(a, \lambda) = N cos^2(a - \lambda), \Rightarrow < A > = \int_{-\pi/2}^{+\pi/2} N cos^2(a - \lambda)\, d\lambda/\pi = (N/\pi)\, [(a - \lambda)/2 + sin2\,(a - \lambda)/4]$$

Evaluate at limits to get $< A > = N/2 = 0.5N.$ *used* $\varrho(\lambda) = 1/\pi,$ *and* the sine contribution drops out at end points. [A “normalizer” N was added just in case we wish to modify all results at the end.(for example getting a better fit to the QM result using $N = \sqrt{2}$ $--$ *a fudge*)]. But we really should be using just N = 1.

So now we can evaluate the correlation of a and b using the standard LHV form Eqn. 1.

$$P(V_a V_b) =< AB >= \int_{-\pi/2}^{+\pi/2} (d\lambda/\pi)\, cos^2(a - \lambda)\, cos^2(b - \lambda)\, N^2 = \int_{-\pi/2}^{+\pi/2} (d\lambda/\pi)[(cos(a - \lambda) cos(b - \lambda)]^2 N^2$$

At first this form looks like another convolution is needed (Figure C). But , it can also be done just using calculus. That is: hint sketch:

Expand $(cos\, x\, cos\, y)^2 = (0.5\, cos(x + y) + 0.5\, cos\,(x - y))^2$, *and use* $\int cos^2 z\, dz = z/2 + sin\, 2z\, /4.$

The result is $P(V_a V_b) =< AB >= 1/8 + cos^2\,(a - b)/4.$ $\neq QM : cos^2(a - b)/2\,!$ Eqn C1.

We might kludge this up by using $N^2 = 2 \rightarrow < AB > = (1/4 + cos^2(a - b)/2\,)$

But, it is still a poor fit because of the 1/8th or ¼ offset value. We actually did much better using what seemed to be silly rectangles in HV-A. Why the offset? Because the lambda domain for integration is only 180 degree wide which is also the domain of each cosine squared function -- there just isn't much room-- it is all used up and some overlap is guaranteed. A minimum overlap may result from a at -45 degrees and b at +45 degrees where we have overlaps on both sides of functions A and B.

[As a quick check, for total overlap: (compare with Eqn B4)

$\alpha = 0\,(max),\ 1/8 + cos^2(0)/4 = 3/8,\ also = \int cos^4\lambda d\lambda = (3/\pi 8)(\pi/2 -- \pi/2) = 3/8.$ Agrees.

Section D: A Better Bell Test:

a). Coincidence for V and H polarizations together

In the older Bell Tests, we only recorded verticals "V and V" correlations and threw away all horizontas, H (we just didn't bother to measure them). The full Figure- **D** tests below are now preferred over the older Bell test.

[Look at coincidence monitor CM for a pair (D- D+) in Figure **D**].

The previous quantum mechanics

$result\ was:\ P_{VV}(a,b) = P_{HH}(a,b) = (1/2)cos^2(a-b), \quad [eqn.\,3\ above]$

But what about the probabilities of coincidence detections for VH and HV , we will need this.

$P_{VH}(a,b) = P_{HV}(a,b) = |<V_a|\ <H_b|\,|\Psi_{DC}>|^2, \quad |\psi_{EPR}> = (1/\sqrt{2}\ [\,|V>|V> + |H>|H>]$

And with a 45 degree laser polarization coming into a nonlinear crystal, EPR = DC. As Before:

$|V_a> = cos\,a|V> - sin\,a|H>,\ and \qquad |H_a> = sin\,a\,|V> + cos\,a\,|H>.$

Carefully picking off the sines and cosines (e.g.,

$<V_a|$V> = <sin a V|V> = sin a <V|V>= sin a), we then get:

$P_{VH}(a,b) = (1/2)[-\,cos\,a\,sin\,b + sin\,a\,cos\,b]^2 = [sin(a-b)]^2/2$

And: $$P_{HV}(a,b) = sin^2\alpha/2 \qquad\qquad Eqn\ D\,1.$$

b) "CHSH": [for John Clauser, Michael Horne, Abner Shimony, Richard Holt, 1969.]

The CHSH HV test [Figure **D**] again uses entangled photons with each photon encountering a two channel polarizer with a total of four outputs (V H left and V H right) instead of the older Bell two detector tests with single channel analyzers. This test has advantages over older Bell; for example not throwing away horizontal H counts but including them in statistics. Counts are recorded for coincidences : N++, N+- , N-+, and N-- (VV, VH, HV, HH).

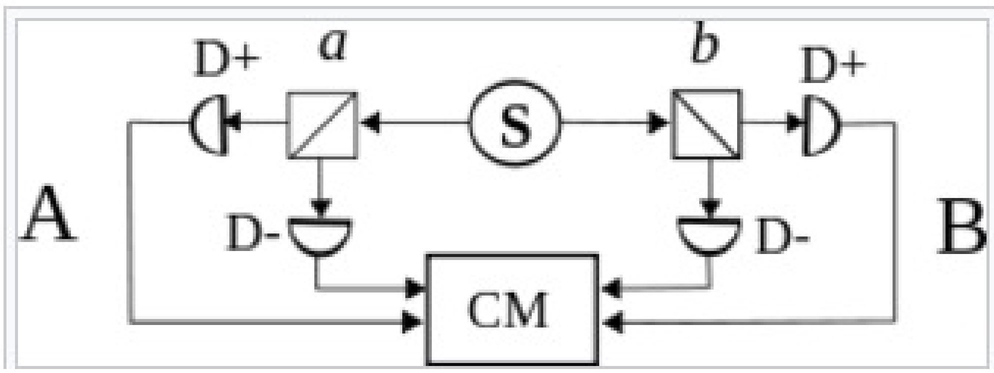
A first experiment begins with angles a and b and then one angle is changed (from b to b') followed by changing a to a' (pair a',b) and finally both angles changed together to pair a'b'. Then statistics are gathered to get an "experimental estimate" E and a special test metric S:

$E = [N_{++} + N_{--} - (N_{+-} + N_{-+})\ ] / \Sigma N's,\ And:$

$$S = E(a,b) - E(a,b') + E(a'b) + E(a'b') \qquad \textbf{Eqn. D 2.}$$

Hidden variables then predict bounds in the form of an inequality:

$|S|_{HV} \le 2,\quad But\ Quantum\ mechanics\ gives\ a\ higher\ range\ \ |S|_{QM} \le 2\sqrt{2}$



<u>Figure D:</u> (Above) **CHSH**-Bell test schematic with a two entangled photon source (S), four detectors (D's), two channel polarizers (a,b), and Coincidence Monitor (CM). [Ref. 12]. This setup is now more popular than Bell.

As hinted by Figure **A**, optimal LHV results may select a set of special test angles separated by 22.5 degrees and angle a'b'=67.5 degrees

$[order\ a' = 0 = V_o,\ b = \pi/8,\ a = 2\pi/8,\ b' = 3\pi/8 = set\ \Phi]$.

This special choice of angles (set "Phi" or similar) was also tested by <u>Aspect</u> [Ref. **14**] --sometimes also called the Bell test angles. They give the highest E(a,b) values for LHV and QM.

From equations 3, D 1, D2 $\{i.e.,\ cos^2\alpha/2,\ sin^2\alpha/2\ and\ E(\alpha)_{QM} = cos\ 2\alpha\}$

$E(a,b) \equiv P_{VV}(a,b) + P_{HH}(a,b) - P_{HV}(a,b) - P_{VH}(a,b) = (1/2)[(1+1)cos^2\alpha - (1+1)sin^2\alpha]$

$But,\ cos^2\alpha - sin^2\alpha = cos(2\alpha).\ And\ for\ special\ case\ \alpha = \pi/8,\ cos(\alpha) = cos(2\pi/8) = 1/\sqrt{2}$ =0.707.

{Notice that $E(a,b)_{QM} = cos(2\alpha)\ is\ a\ cosine\ curve\ from\ +1\ at\ 0\ to\ -1\ at\ \pi/2.$}

The sequential calculations for CHSH first calculate probabilities like P(vv) in Eqn A1 and equation D1. Then find "estimate" E(a,b) in terms of four P's (like D5). And then the S metric

(Eqn D2) using four E values. It sounds hard. But it gives the most reliable sets of inequality limits, S.

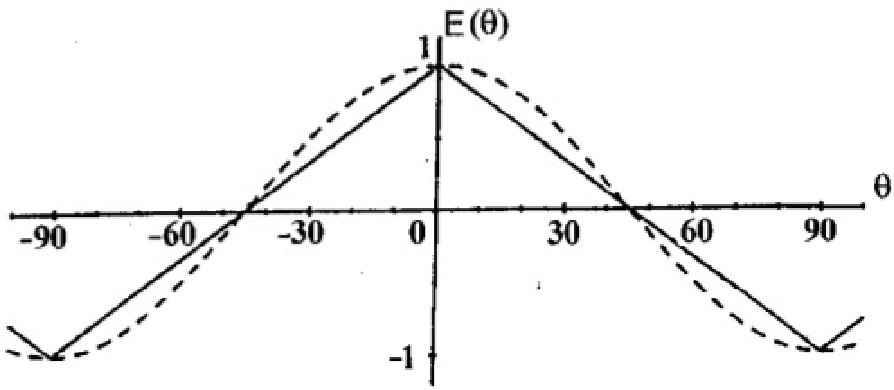


***Figure 3** - Polarisation correlation coefficient, as a function of the relative orientation of the polarisers: (i) Dotted line : Quantum Mechanical prediction ; (ii) solid line : the naive model.*

Figure E: (Figure 3 from Reference [5]) -- Has the same shape as Figure A but now with y-axis = E(alpha) = **E(a,b)** = "Experimental coincidence estimate" instead of the P(VV) of just one of its components. This commonly seen graph has a range double that of Fig. **A**. $E(\alpha)_{HV} = 1 - 4|(a,b)|/\pi \; versus \; E(a,b)_{QM} = cos(2\alpha)$.

This is different from the cosine curve in Figure **A** which was only for P_V V (range 0 to +0.5) while the E plot is stretched up from Fig. **A**. But for the definition of example HV-A with the convolution of rectangles for "joint snap to" output P_v v as the only thing of interest. Now we have broadened our interest beyond just P_vv.

$Also\ E(a', b') = cos2(3\pi/8) = -0.707.\ So,\ [Eqn\ D2]:\ S_{QM} = 3/\sqrt{2} - -1/\sqrt{2} = 2\sqrt{2} = 2.828$

And the 1982 measurement by Alain Aspect [14] gave **S~ 2.7**, pretty close and well above $S_{HV} \le 2.0$. (implying that QM is not local-real [hence non-local]).

OK, so we can calculate the CSCH inequality limit for QM.
Can we find a **local** hidden value comparison or inequality limit for S?

Well, **abstractly**, it is very easy [11]: for convenience, first abbreviate with a single letter and for each lambda and each test where each letter can only be 1 or -1:

$A(a,\lambda) = just\ A,\ value\ B\ is\ \pm 1,\ B + B'\ or\ B - B'\ has\ to\ be\ 0\ or\ \pm 2.\ Then\ examine\ the\ 4\ quantities$

(like the metric S = S(E's) in equation **D2** above)

$AB + AB' + A'B - A'B' = A(B + B') + A'(B - B') = A(2) + A'(0)\ or\ A(0) + A'(2) \le 2$

This is the inequality for CHSH hidden variables ("Experimental estimates E). -- Pretty Clever, and again QM can do better (above 2.0) in violation of LHV's.

<u>c). A Concrete Local Hidden Variable for CHSH</u> : (continue deriving the Cosine versus sloped line plots somewhat like HV-A but now for estimate E). Let us evaluate **t<u>he best HV estimate E(a,b)</u>** <u>plot</u> (using the A(a) = Π rectangles as in Fig **C** and convolution). This is still similar to Fig C but now having a vertical axis E(a,b) where **E** depends on **all four** probabilities PVV + PHH - PVH - PHV (for aligned vs. unaligned coincidences). But so far we only knew about the PVV term in HV_A -- so how can one calculate the other probabilities? There is a very clever trick found in AJP [pg 909, ref. [**<u>2</u>**][15] . We can state two old terms familiar A and B as separate functions of a new A' and B' for use in calculating estimate E(b-a) such that:

$$P_{VV}(a,b) = \int A(\lambda,a)\cdot B(\lambda,b)\,\varrho(\lambda)d\lambda = \int 0.5[1 \pm A'(\lambda,a)]\cdot 0.5[1 \pm B'(\lambda,b)]\,\varrho(\lambda)d\lambda \qquad \textbf{Eqn. D 3}$$

with just a change in signs for the other combinations: **VV(++), HH(--), HV(-+), VH(+-).** And then the resulting summed estimate is a pleasant form with primes $'$

$$E(a,b) = \int A'(\lambda,a)B'(\lambda,b)\,\varrho(\lambda)d\lambda \qquad \textbf{Eqn. D 4.}$$

If we know the PVV, then we exploit this symmetry of signs to now deduce the other three expressions. Summing up all the possible combinations of integrands above (and using the definition of E with its own + for V's and - for H's) of the integrands on the right side of the equation above will collapse this apparent complexity into just the simple **A'B'** estimate on the left. The old familiar A B terms now look like:
$A = \Pi = [1 + A']/2 \Rightarrow A' = 2A - 1$ (*for all the vertical's, V*), *and* $[1 - A']/2$ represents horizontals H.

Now recall that the old A and B functions of eqn **A0** were just rectangles with height = 1 (like in Fig. C). BUT notice that the rectangle A only takes up 90 degrees on the lambda axis while the domain is 180 degrees. So the number 1 represents a flat constant altitude plotted all across the domain. And [1-A']/2 has a notch down to zero below the line ½ , or $1/2\,(1 - \Pi)$, or this time we look for **<u>W = Non-Overlap!</u>** -- only the domain <u>outside</u> of the rectangular notch counts. So for PVV we are convoluting

$$\Pi * \Pi,\ for\ P_{HH}\ we\ find\ (1 - \Pi) * (1 - \Pi).\quad For\ P_{HV} = P_{VH},\ find\ \Pi * (1 - \Pi).$$

Again, the domain of the integral is $\pi/2 - (-\pi/2) = \pi,\ so\ \varrho(\lambda) = 1/\pi$ for all the integrals.
The 4 relevant overlap equations to sum up for (b-a) angle are:

$$E(\alpha) = (1/\pi)[(\pi/2 - \alpha/\pi) + (\pi/2 - \alpha/\pi) + (0 - \alpha/\pi) + (0 - \alpha/\pi)] = [1 - 4\alpha/\pi] \quad \textbf{-- Eqn. D5.}$$

a down-sloping line for this LHV (shown in Figure **<u>E</u>**) against $E(\alpha)_{QM} = cos(2\alpha)$.

Non-Local:

For reference, the most popular "non-local hidden variable" is the de Broglie-Bohm "position" x(t) along with the velocity of a moving particle. Remember that Copenhagen doesn't believe in the existence of trajectories, but dBB ~ QM works [9]! It is equivalent to usual QM but in a different form and different interpretation. In Bohm theory, "the non-local correlations are a consequence of the non-local "quantum potential," which exerts suitable torque on the particles leading to experimental results compliant with quantum mechanics [8]." dBB is not very popular, but it was intended as just an example of non-local hidden variable theory -- and as a counter-example, it revealed an error in von Neumann's "proof" of no hidden variables.

A separate class of non-local hidden variables was introduced by **Leggett** in 2003 along with a new inequality for testing. Assumptions are 1: realism (pre-existing properties independent of measurement) e.g., polarization u for A and v for B, 2: "physical states are statistical mixtures of sub-ensembles with definite polarization where" 3: Malus' law cosine projections apply for each sub-ensemble. A new nonlocal parameter, η, is introduced for arranging measurement settings across space-like separation of detectors A and B (often called "Alice" and "Bob"). Large statistics averagings are arranged (or contrived) to satisfy some QM expectation values. The contrivance is complex, so as just a partial sketch: the distribution for $\lambda \in [0,1]$ is decomposed into two parts for A at value L and into 3 different parts for B. For example:

$$A = A(a,u,\lambda) = +1 \, for \, \lambda \in [0,L), \; -1 \, for \, \lambda \in [L,1], \; L = 0.5(1+u \cdot a), \; B = B(a,b,u,v,\lambda)$$

That is, Bob does all the statistical contriving [8], and he knows about Alice's settings "outside of space-time".

Actual testing of Leggett versus QM result in plots somewhat like Fig A that differ significantly for tilt setting difference $\alpha \sim \pm 30^o$ showing that "non-signalling correlations" don't work.

Terms:

Definitions of reality, locality, and causality [11].

Reality or realism is the belief that the wave function (exists prior to detection) is a physical field, like an electric field, rather than just information, like a weather forecast. ["outcomes of measurements that are not performed are just as real as those of measurements that were performed" -- counterfactual definiteness].

Locality means that the wave function is a function of only the local position and time, and that the value of the wave function at a given point does not depend on anything else, specifically not on anything somewhere else [an object is only directly influenced by its immediate surroundings]

Causality means that the value of the wave function at a particular point cannot change because of something that happens outside of the past light cone of that point.

With these definitions, it is easy to see (based on the experiments that disagree with Bell's inequality) that physics cannot satisfy all three of these conditions.
Add: "What we cannot exclude, as with any experiment, is the possibility (of) an earlier common cause in the overlap of the backward light cones of the two events."

The existence of "non-locality" (NL) was already present in the original Copenhagen concept of reduction (R) or **"wave function collapse"** [17]. This is now enhanced by "EPR + Bell $\Longrightarrow$ NL." Copenhagen QM also advocates that a state vector applies to each single system. But many now believe that it only applies to a statistical ensemble to give a "forecast". This problem is shown simply in the Young two-slit interference experiments. Classically, light passes through both slits to form a pattern on a screen. This is like $\psi \sim E,\ \psi^*\psi \sim E^*E$ = Energy of the electric field, profile for $E^2(x,y)$ --[We learned in EM [electricity and magnetism] that the energy stored in an electric field distribution is proportional to the square of E]. Photons go where the energy density is, but we can only see this using a statistical ensemble over many photon collapses. Random selectings follow the EM energy distribution on the screen, and that distribution exactly matches the energy distribution (or probability distribution). But how can each photon collapse know where to unless each single system already knows the propensity pattern...?

What does QFT add? Well, it doesn't make any prediction on single events but only on the ensembles. And QFT is highly local ! But the idea of non-locality was intended to apply for each single pair of events. However any clear interpretation of QM or QFT exists, so the issues cannot be clarified at this time.

Conclusions:

In the mathematics above, we have largely bypassed the vast subject of generalized "Bell inequalities" tests. Instead, we have addressed the continuous graphs of QM correlations versus local hidden variables models as a function of two polarizer-detectors having tilt angle differences, $\alpha = (b-a)$ (e.g., Figure A with QM versus concrete example "HV-A" with a modestly good fit). We have derived the fundamental quantum mechanical correlation equation: $P(VV) = cos^2\alpha/2$, and we used this result to attempt another LHV for A and B as cosine-squares (HV-C with a poorer fit than HV-A). Then we came up with an example HV-B that unintentionally ended up being the same as QM. But thought revealed that it wasn't Local. However, it might seem to be a counterexample to saying "no hidden variables."

Next we examined CHSH type tests for "experimental estimates" $E(a,b)_{QM} = cos2(b-a)$. Calculating $E(a,b)_{HV}$ required convolutions. Finally, the values from the key graphs (Fig A and Fig. E) could allow us to find the final metric S and show its violation $S_{QM} > S_{HV}\ using\ angle\ set\ \Phi$ for optimal input test angles. All examples shown here have function plots of $\alpha = (b-a)$ angle differences.

Discussion:

A general goal of my papers is to study a subject and then present it more simply for easier understanding. It would appear that I didn't succeed very well with this topic. It was hard to locate literature to find the math of the topic choices here. And it was difficult to see how to understand that math intuitively (and of course the interpretation of quantum mechanics has always been tricky with little agreement between experts). Many intuitive presentations of entanglement seem to use analogies that really don't apply very well. Sorry.

A great majority of current journal articles seem to solidly support the conclusion that Bell-theory implies **non-locality** ("spooky action at distance") and the impossibility of Local hidden variables (LHV's). Yet, a recent survey on physicist's beliefs express residual doubt [survey, 10] at about one-third of physicists. Additional beliefs from the survey suggest that 2/3rds believe that true randomness is inherent in QM detections, 2/3rds believe that we need to have interpretations of quantum mechanics; yet ¾'s believe either in Copenhagen or simply don't care (which is another aspect of Copenhagen).

Possible explanations for non-locality refer to hidden sub-quantum-level signalling going freely backwards and forward in time from one detector back to the source and then to the other detector for a joint handshaking. This retrocausality was originally suggested in 1950 by a Parisian physicist, Olivier Costa de Beauregard to help explain EPR. This **"Parisian zigzag"** idea is still studied (and this was before Kastner, Cramer, and Aharonov).

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