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ABSTRACT

In the mid-nineteenth century George Boole formulated his 'conditions of possible experience'. These are equations and inequalities that the relative frequencies of (logically connected) events must satisfy. Some of Boole's conditions have been rediscovered in more recent years by physicists, including Bell inequalities, Clauser Horne inequalities, and many others. In this paper, the nature of Boole's conditions and their relation to propositional logic is explained, and the puzzle associated with their violation by quantum frequencies is investigated in relation to a variety of approaches to the interpretation of quantum mechanics.

* While preparing this paper for publication I have learnt of the untimely death of Professor J. S. Bell, and I wish to dedicate the paper to his memory. This research was undertaken while I spent a sabbatical leave at Wolfson College, and the History and Philosophy of Science Department at the University of Cambridge. I would like to thank Michael Redhead and Jeremy Butterfield for their hospitality and for helpful discussions. A first draft of this paper has been distributed among the participants of the conference 'Einstein in Context' which was held in Israel, in April 1990. I have benefited from the comments of many colleagues. I would like to thank in particular Arthur Fine who enlightened me on the prism models, David Albert, Maya Bar-Hillel, Yemima Ben-Menachem, Mara Beller, Simon Saunders, and Mark Steiner. This research is partially supported by the Edelstein Center for the History and Philosophy of Science at the Hebrew University.

> These . . . may be termed conditions of possible experience. When satisfied they indicate that the data *may* have, when not satisfied they indicate that the data *cannot* have resulted from an actual observation.
>
> George Boole [1862]

I INTRODUCTION

An interpretation of quantum mechanics (q.m.) naturally involves ideas about probability. There are two levels at which this concept enters the discussion. On the first, more superficial, level there is the problem of determinism. As is well known, given an initial state of a physical system, q.m. predicts only a probability distribution over the values of future observations. The question is whether this is, in some sense, a limitation of principle or, whether a better theory can provide more definite predictions.

On the second, deeper, level comes the realization that the quantum concept of probability is formally incompatible with the classical one.[1] As we shall see later this realization bears directly on the problem of *causality*. Now the question is whether microphysical events can be made consistent with *any* (deterministic or otherwise) causal explanation. It is precisely this shift in emphasis which brings out the revolutionary character of quantum physics.

In the early days of quantum theory, up until the 1960s, foundational disputes tended to concentrate on determinism and related issues. Everyone realized of course that the calculation of probabilities in q.m. is peculiar in the sense that it involves complex valued functions. But only a few of the more mathematically oriented physicists and some mathematicians noticed that this calculation procedure yields results which are formally incompatible with classical probability. Even fewer took this fact to have fundamental importance.[2]

The peculiar nature of quantum probability had been recognized gradually, the first indication being probably the case of quantum statistics. If we take Planck's [1990] paper on the black body radiation out of historical context, and ask what is *intrinsically* puzzling about it, then the answer is not that light is assumed to be quantized. There is nothing intrinsically puzzling about a corpuscular theory of light, a perfectly classical idea. Rather it is the way in which photons (light particles) are distributed which is puzzling. But this fact was not explicit in Planck's derivation, and was discovered a quarter of a century later by Bose [1924]. Still the Bose–Einstein distribution (or the

1 By 'classical probability' I mean the theory which appears in all standard textbooks, and whose axiomatic formulation is due to Kolmogorov. As we shall see subsequently, the problems we are dealing with arise quite independently of the interpretation of these axioms and of the meaning of the term 'probability'.

2 An exceptional attitude was already expressed by Shrödinger in 1935: 'At no moment in time is there a collective distribution of classical states which would be in agreement with the sum total of quantum mechanical predictions' (Shrödinger [1935]) I would like to thank Y. Ben-Menachem for calling my attention to this reference.

Fermi–Dirac distribution which corresponds *e.g.* with electron gas) does not contradict the principles of classical probability; it is just highly counter-intuitive.[3]

The difference between classical and quantum probability was clearly revealed in Wigner [1932]. The author attempted to construct a distribution on phase space, which would be the quantum analogue of the distribution function of classical statistical mechanics. He succeeded to define a real valued function on phase space, nowadays called the Wigner distribution, from which the expectation values of quantum mechanical observables could be recovered. But Wigner's 'distribution' has negative values and cannot be made non-negative. The importance of Wigner's discovery for foundational problems was not recognized until much later.[4]

A different early perspective can be found in Birkhoff and von Neumann [1936], seminal paper on quantum logic. They argued for a radical thesis, namely, that microphysical events do not conform to the rules which we usually associate with an algebra of events (Boolean algebra), but rather with a different set of rules which they called 'quantum logic'. Consequently, probability measures defined on a quantum logic violate the axioms of classical probability theory. Again this contribution was by and large ignored until much later.[5] Thus, as late as 1949, Max Born, who is credited with the discovery of the statistical interpretation of the wave function, hardly mentioned the difference between classical and quantum probability when he summarized his philosophical views.

This difference becomes most transparent in the path integral formalism due to Feynman [1948]. There is a formal similarity between the Feynman path integral and the Wiener integral of the Brownian motion, a similarity that was recognized immediately.[6] Both integrals represent summation over the continuous paths that a particle may take (the path sample space). But while the Wiener integral is an integral with respect to a well-defined probability measure, the Feynman integral is, mathematically speaking, not an integral at all, not even with respect to a complex valued measure.[7] The transition probabilities, which are calculated from the Feynman path integral by taking

3 This is at least the way many probability theorists felt, for example W. Feller: 'The appropriate or "natural" probability distribution seemed perfectly clear to everyone and has been accepted without hesitation by physicists. It turned out, however, that physical particles are not trained in human common sense, and the "natural" (or Boltzmann) distribution has to be given up for the Einstein-Bose distribution in some cases, for the Fermi-Dirac distribution in others. No intuitive argument has been offered why photons should behave differently from protons and why they do not obey the "a-priori" laws' (Feller [1957], p. 5).

4 Moyal [1949]. See also Bartlett [1945].

5 Interest in quantum logic has revived with the emergence of the axiomatic approach to q.m.: Mackey [1963]. The probabilistic structure associated with quantum logic has been completely identified, due to a deep theorem by Gleason [1957].

6 The similarity is not so surprising given that the Shrödinger equation is a diffusion equation, whose 'diffusion coefficient' is a pure imaginary number. See Kac [1959].

7 See, for example, Cameron [1960].

the square of its absolute value, are even further removed from a classical probabilistic picture. Physicists say that the various possible paths 'interfere'.

Theory aside, the crucial question is what are the consequences of these highly abstract observations to experience. The answer is quite obvious: the difference between classical and quantum probability is manifested in the phenomena of interference. But this answer is unsatisfactory for 'interference' is itself a theoretical, or at least a theoretically loaded term.

In the classical theory of probability, the observational counterparts of the theoretical concept 'probability distribution' are the relative frequencies. In other words, as far as repeatable (independent or exchangeable) events are concerned, probability is manifested in frequency.[8] My first aim in this paper is to analyse the phenomena of interference in terms of relative frequencies.

Surprisingly, the tools for such an analysis were developed, independently of physics, over the last 140 years, beginning with George Boole [1862]. Boole's research problem in this context can be phrased in modern terminology as follows: we are given a set of rational numbers $p_1, p_2, \ldots, p_n$ which represent the relative frequencies of n logically connected events. The problem is to specify necessary and sufficient conditions that these numbers can be realized as probabilities in *some* probability space. In other words, to establish the conditions under which there exist some classical probability space, n events $E_1, \ldots E_n$ in that space which manifest the aforementioned logical relations, such that p_i=probability (E_i) for $i=1, 2, \ldots, n$.

The conditions in question were called by Boole 'conditions of possible experience'. They depend on the logical relations among the events and always take the form of linear inequalities and equalities in the numbers $p_1, \ldots, p_n$.

In quantum mechanics[9] we do not and cannot have a classical probability distribution from which to recover the expectation values of all the observables. This means that the relative frequencies of microphysical events (which are usually measured in several distinct samples) sometimes violate some of Boole's conditions associated with these events. This is what 'interference' is, and as far as the phenomenon itself is concerned, this is all that it can be.

As we shall see, Boole's conditions of possible experience can be derived from very elementary assumptions, either those of probability theory or alternatively those of propositional logic. Hence the puzzling aspects of q.m. reside in the phenomena themselves and not just in the theory which predicts them. To a large extent, therefore, we are dealing here with a puzzle which does not depend on the details of a complex physical theory. Even if the theory changes in the future, the puzzle will probably remain.

[8] This is just the law of large numbers. Either the version due to de Finetti, which concerns exchangeable events, and thus preferred by the subjectivists, or the usual version which concerns independent events.

[9] The connection between Boole's problem and quantum mechanics was first indicated in Pitowsky [1989a].

This point can be illustrated from a different angle. Quantum mechanics is considered to be a paradigm case of a scientific revolution. But why? Really, what is so revolutionary about it? This seemingly naïve question turns out to be difficult to answer. If we look back to the disputes among the old masters, or alternatively search in the standard textbooks, we shall find only partial answers. The difference between classical and quantum physics is usually described in the language and terminology of quantum mechanics itself ('wave particle duality', 'collapse of the wave packet', 'uncertainty relations', and the like). In the worst cases, it is described in terms of an interpretation of that theory ('complementarity').

There is an sense of question begging in these answers. A scientific revolution, if it is to deserve this dramatic caption, must have an experimental aspect to it. It is precisely this aspect which forces us to consider radical new theories in the first place. If this logical (though perhaps not historical) order of things is to be preserved, there must be a way to *describe* the phenomena which is independent of the theory from which it is subsequently deduced. In the case of q.m., Boole's conditions provide an appropriate language for such a description.

For these reasons I have decided to adopt the 'bottom-up' approach, begin with a description of the phenomena, move to the theory, and conclude with interpretations. More specifically I attempt to answer three questions:

WHAT precisely is it about microphysical phenomena that is different from classical phenomena?
HOW is this difference incorporated into quantum theory?
WHY is it that microphysical phenomena and classical phenomena differ in the way they do?

In Section 4 I shall provide an answer to the first question, expressed in terms of Boole's conditions of possible experience. The derivation and meaning of these conditions is discussed in Sections 2 and 3. The second question is really a technical one and I shall touch upon it briefly, in a non-technical manner, in Section 5. The third question is the problem of interpretation. A priori it is not clear that an answer to such a question is required, nor is it clear what kind of answer can qualify as a reasonable one. I shall nevertheless argue, in Section 6, that in the present context the WHY question does make sense and that there are good *philosophical* (as opposed to scientific) grounds to look for an answer. A few alternative interpretations are suggested in the rest of the paper. I have made the effort to present the material in the least technical way possible. Some further, slightly more technical material is given in the three appendices.

The method adopted in this paper seems suspiciously empiricistic, yet the motivation behind it is not. Unlike the empiricist I am not at all concerned here with justification, induction, or the analysis of theory acceptance. I merely attempt to describe:

> We must do away with all *explanation*, and description alone must take its place. And this description gets its light, that is to say its purpose from the philosophical problems. . . . The problems are solved not by giving new information, but by arranging what we have always known. Philosophy is the battle against the bewitchment of our intelligence by means of language. (Wittgenstein [1978], p. 109)

2 BOOLE'S CONDITIONS OF POSSIBLE EXPERIENCE AND THEIR DERIVATION

George Boole is best known as one of the fathers of modern logic. Somewhat less known in his work on the theory of probability, most of it published in his classical book, Boole [1854]. About ten years after the publication on this influential treatise, Boole arrived at a clearer formulation of a problem he had considered to be of central importance for the theory of probability:[10]

> We are now able to explain more clearly the nature of the analytical investigation which will follow. Let $p_1, p_2, \ldots, p_n$ represent the probabilities given in the data. As these will in general not be the probabilities of unconnected events, they will be subject to other conditions than that of being positive proper fractions, vis. to other conditions beside
>
> $$p_1 \geqslant 0,\ p_2 \geqslant 0,\ \ldots,\ p_n \geqslant 0$$
>
> $$p_1 \leqslant 1,\ p_2 \leqslant 1,\ \ldots,\ p_n \leqslant 1$$
>
> Those other conditions will, as will hereafter be shown, be capable of expressions by equations or inequations reducible to the general form
>
> $$a_1 p_1 + a_2 p_2 + \ \ldots\ + a_n p_n + a \geqslant 0$$
>
> $a_1, a_2, \ldots, a_n, a$, being numerical constants which differ for the different conditions in question. These together with the former, may be termed conditions of possible experience. When satisfied they indicate that the data *may* have, when not satisfied they indicate that the data *cannot* have resulted from an actual observation.

A few words of clarification. What Boole means here by 'probability' is relative frequency in a finite sample. As we shall see below, the conditions in question apply to the concept of probability quite independently of the meaning attached to this term. Boole's problem is simple: we are given rational numbers which indicate the relative frequencies of certain events. If no logical relations obtain among the events, then the only constraints imposed on these numbers are that they each be non-negative and less than one. If however, the

[10] More on Boole's conditions of possible experience and their relations to his more general concerns will be found in Hailperin [1986].

events are logically interconnected, there are further equalities or inequalities that obtain among the numbers. The problem thus is to determine the numerical relations among frequencies, in terms of equalities and inequalities, which are induced by a set of logical relations among the events. The equalities and inequalities are called 'conditions of possible experience'. A few examples:

(a) Suppose that p is the relative frequency of the event E and q is the relative frequency of the event $E \cap E'$ then $q \leqslant p$ or

$$p - q \geqslant 0$$

(b) If p_1, p_2 are the probabilities of the events E_1, E_2, respectively, and q_1, q_2 are the probabilities of $E_1 \cap E_2$ and $E_1 \cup E_2$, respectively, then $p_1 + p_2 = q_1 + q_2$, or

$$p_1 + p_2 - q_1 - q_2 = 0$$

This identity is commonly taken as one of the axioms of the calculus of probability.

(c) If p is the probability of the event E and q the probability of its complement $\bar{E}$, then $p + q = 1$, or

$$p + q - 1 = 0$$

which is another axiom of the calculus of probability.

(d) If p_1, p_2 are the relative frequencies of E_1, E_2, respectively, and q the relative frequency of $E_1 \cap E_2$, then $p_1 + p_2 - q \leqslant 1$, or:

$$-p_1 - p_2 + q + 1 \geqslant 0$$

From a mathematical point of view Boole's achievement lies in the realization that all the 'conditions of possible experience' are *linear* in the probabilities. In other words, the inequalities and equalities never involve expressions such as p^2 or 2^q when p, q are probabilities. Moreover, given a finite set of events, with (obviously) finitely many logical relations obtaining among them, there is only a finite set of conditions which hold. To be more precise, there is a finite set of equalities and inequalities, from which all other valid conditions logically follow. This means that there is an algorithm which—whenever given a set of events and their logical relations as input—produces the relevant Boole's conditions (in the form of a finite set of linear inequalities and equalities) as output. Although Boole's conditions are decidable, short instances of the problem may require an extremely long time to calculate.

In the rest of this section I shall indicate how to derive Boole's conditions.[11] I shall do so with the aid of a few examples. The outline of the general algorithm is given in Appendix 1. This method is important to our concern because it reveals the relationship between Boole's problem and propositional logic.

[11] The method presented below is in Pitowsky [1989b]. Mathematical aspects of Boole's problem and related issues appear in Pitowsky [1991].

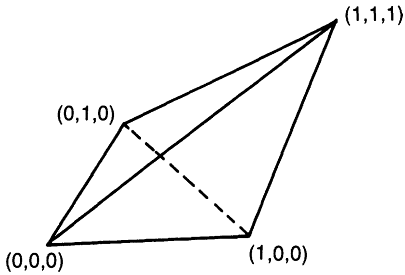

FIGURE 1

As a first example consider two events E_1, E_2 with relative frequencies p_1, p_2 and let p_{12} denote the frequency of the joint $E_1 \cap E_2$. Our purpose is to determine Boole's conditions on the numbers p_1, p_2, p_{12}. Clearly we should have:

$$p_{12} \geqslant 0,\ p_1 \geqslant p_{12},\ p_2 \geqslant p_{12} \tag{1}$$

Also the frequency of $E_1 \cup E_2$ is $p_1 + p_2 - p_{12}$, so we must have

$$p_1 + p_2 - p_{12} \leqslant 1 \tag{2}$$

Inequalities (1) (2) together are necessary and sufficient for the rational numbers p_1, p_2, p_{12} to be the frequencies of two events and their joint. In other words, they are Boole's conditions in this case.

These simple observations have a direct geometrical representation: Consider the three-dimensional (real) space and in it the set of all vectors of the form (p_1, p_2, p_{12}), where p_1, p_2, p_{12} satisfy inequalities (1) and (2). This set is a convex polytope (Figure 1) whose vertices are (0,0,0), (1,0,0), (0,1,0), (1,1,1). The vertices represent extreme cases: (0,0,0) is the case where $p_1 = p_2 = 0$ and of course the frequency of the joint p_{12} is then zero too; (1,0,0) is the case where $p_1 = 1$ while $p_2 = 0$, which entails that $p_{12} = 0$, and so on.

Every convex polytope in a Euclidean space has a dual description, either in terms of its vertices or in terms of its facets. Under the first description, a given vector is an element of the polytope if and only if it can be represented as a convex combination (weighted average) of the vertices. Under the second description, a vector is an element of the polytope if and only if its coordinates satisfy a set of linear inequalities[12] which represent the 'half spaces' whose intersection is polytope. The existence of such a dual description for every convex polytope is known as the Weyl–Minkowski theorem.

In the specific case above, our starting point has been the second type of description. We have derived Boole's conditions (inequalities 1,2) first, and subsequently found the vertices. If we adopt the subjective approach to probability we shall obtain the same result, in a reverse order.

[12] If the polytope is full-dimensional, *i.e.* has non-empty interior, then we have inequalities only. If however, the polytope is confined to an affine hyperplane, we have equalities as well.

Given two propositions—a_1 'it will rain in Paris tomorrow' and a_2 'it will rain in Madrid tomorrow'—we consider the truth table for a_1, a_2 and a_1 & a_2 (0 stands for 'false', 1 stands for 'true'):

a_1	a_2	a_1 & a_2
0	0	0
1	0	0
0	1	0
1	1	1

The rows of the table, if looked at as vectors in a three-dimensional space, are just the vertices of our polytope. The rows also represent the four logical possibilities: that it will rain neither in Paris nor in Madrid tomorrow, that it will rain in Paris but not Madrid tomorrow, etc. Suppose that we were to bet on each one of the four possibilities, then we would choose four non-negative numbers λ_1, λ_2, λ_3, λ_4 such that λ_1 is the subjective probability assigned to 'it will rain neither in Paris nor in Madrid tomorrow', λ_2 the probability assigned to the second row in the truth table, and so on. Since there are only four possibilities, and since they are mutually incompatible, coherency entails that $\lambda_1+\lambda_2+\lambda_3+\lambda_4=1$.

Consider the vector sum:

$$(p_1,p_2,p_{12})=\lambda_1(0,0,0)+\lambda_2(1,0,0)+\lambda_3(0,1,0)+\lambda_4(1,1,1)=(\lambda_2+\lambda_4,\lambda_3+\lambda_4,\lambda_4)$$

Here $p_1=\lambda_2+\lambda_4$ is the subjective probability assigned to the proposition a_1, that is, 'it will rain in Paris tomorrow'; $p_2=\lambda_3+\lambda_4$ is the probability of 'it will rain in Madrid tomorrow' and p_{12} is the probability of the joint. Since (p_1,p_2,p_{12}) is a weighted average of the vertices it is an element of the polytope and thus necessarily satisfies Boole's conditions. In other words *in the subjective conception of probability, Boole's conditions are just the conditions of coherency.*

These considerations can be generalized easily. Let us take another example, beginning with the subjective view this time. Consider three propositions a_1, a_2, a_3 and their three pair conjunctions a_1 & a_2, a_1 & a_3, a_2 & a_3. There are eight possible truth value assignments to three propositions (and thus also for the conjunctions). These are given by the following table:

a_1	a_2	a_3	a_1 & a_2	a_1 & a_3	a_2 & a_3
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
1	1	0	1	0	0
1	0	1	0	1	0
0	1	1	0	0	1
1	1	1	1	1	1

Take each row in the table as a vector in a six-dimensional real space. There are eight such vectors, and their closed convex hull in the six-dimensional space is a full-dimensional polytope. Given a six-dimensional vector $(p_1, p_2, p_3, p_{12}, p_{13}, p_{23})$, it is an element of the polytope if, and only if, it can be represented as a weighted average of the eight vertices.

Boole's conditions on the probabilities of three events E_1, E_2, E_3 with relative frequencies p_1, p_2, p_3 respectively, and their three pair joints $E_1 \cap E_2$, $E_1 \cap E_3$, $E_2 \cap E_3$, with relative frequencies p_{12}, p_{13}, p_{23}, are just the facet inequalities of the above polytope. Let us attempt a guess. Firstly it is obvious that each pair out of the three events must satisfy the conditions on a pair of events and their joint. So we have for $1 \leqslant i < j \leqslant 3$:

$$p_{ij} \geqslant 0, \qquad p_i \geqslant p_{ij}, \qquad p_j \geqslant p_{ij}, \qquad p_i + p_j - p_{ij} \leqslant 1. \tag{3}$$

But these conditions are not sufficient, we must add to them constraints on all three events and their joints. The relative frequency of $E_1 \cup E_2 \cup E_3$ is greater or equal to $p_1 + p_2 + p_3 - p_{12} - p_{13} - p_{23}$, so

$$p_1 + p_2 + p_3 - p_{12} - p_{13} - p_{23} \leqslant 1. \tag{4}$$

Now if instead of the first event E_1, we substitute its complement $\bar{E}_1$, then we have to replace p_1 by $1 - p_1$, replace p_{12} by $p_2 - p_{12}$, and replace p_{13} by $p_3 - p_{13}$, while p_2, p_3, p_{23} remain intact. Substituting these values into (4) we get

$$p_1 - p_{12} - p_{13} + p_{23} \geqslant 0 \tag{5}$$

and by symmetry

$$\begin{aligned} p_2 - p_{23} - p_{12} + p_{13} &\geqslant 0 \\ p_3 - p_{13} - p_{23} + p_{12} &\geqslant 0 \end{aligned} \tag{6}$$

Satisfying inequalities (3) through (6) is a necessary and sufficient condition that the vector $(p_1, p_2, p_3, p_{12}, p_{13}, p_{23})$ is an element of the polytope. Boole's conditions (4), (5), (6) have a special name in the physics literature; they are called Bell inequalities.[13]

The method illustrated above hints at the general algorithm. Suppose that we are given the probabilities of a number of logically connected events. The logical connections are exhibited in terms of propositional (that is, Boolean) formulas. We write down the truth table for the formulas, take the convex hull of its rows to obtain a polytope. Next we determine the inequalities (and equalities, if any) corresponding to the polytope. These are Boole's conditions.

[13] Bell [1964]. Bell did not use these inequalities explicitly. They appeared first in Wigner [1970]. In these, as in many subsequent papers by physicists, mathematical and physical considerations intermingle in the derivation of the inequalities. The purely probabilistic nature of Bell inequalities has been demonstrated in Fine [1982a] where the sufficiency of these inequalities is also proved. The relations to Boole's problem and the polytope method were developed in Pitowsky [1989a, 1991].

In Appendix 1 the reader can find more details, as well as a formal explanation why this method yields the correct answer.

In the following I shall deal with relative frequencies only. More precisely, with relative frequencies in a *finite* sample. By this I do not intend to imply that probabilities *are* relative frequencies. Regardless of one's view on the meaning of 'probability', in the case of repeatable (exchangeable or independent) events, probability is *manifested* in frequency.

3 CAN BOOLE'S CONDITIONS BE VIOLATED?

One thing should be clear at the outset: *none of Boole's conditions of possible experience can ever be violated when all the relative frequencies involved have been measured in a single sample.* The reason is that such a violation entails a logical contradiction. For example, suppose that we sample at random a hundred balls from an urn. Suppose, moreover that 60 of the balls sampled are red, 75 are wooden and 32 are both red and wooden. We have $p_1 = 0{\cdot}6$, $p_2 = 0{\cdot}75$, $p_{12} = 0{\cdot}32$. But then $p_1 + p_2 - p_{12} > 1$. This clearly represents a logical impossibility, for there must be a ball in the sample (in fact three balls) which is 'red', is 'wooden', but not 'red and wooden'; absurd.

Similar logical absurdities can be derived if we assume a violation of any of the relevant conditions, no matter how complex they appear to be. This is the reason for the title 'conditions of possible experience'. *In case we deal with relative frequencies in a single sample, a violation of any of the relevant Boole's conditions is a logical impossibility.*

But sometimes, for various reasons, we may choose or be forced to measure the relative frequencies of (logically connected) events, in several distinct samples. In this case a violation of Boole's conditions may occur. There are various possible reasons for that, and they are listed below in an increasing order of abstractness:

(a) *Failure of randomness.* Suppose that we have a large urn containing a vast number of balls of various colours sizes and constitutions. We measure the relative frequency of red balls in one sample and obtain a number p_1. Next we measure the relative frequency p_2, of wooden balls *in a second distinct* sample. Finally we measure the relative frequency p_{12} of red wooden balls in a third sample. *If the samples are sufficiently large we still expect Boole's conditions (1), (2) to obtain.* The reason lies with the law of large numbers: with high probability, the relative frequency of red balls in a large random sample, is close in value to the proportion of red balls in the urn. The same is true with respect to the other properties, 'wooden' and 'red and wooden'. Since the proportions of balls *in the urn* (the 'population') satisfy Boole's conditions then so do, with high probability, the numbers p_1, p_2, p_{12}. Still, we should not be surprised perhaps to obtain $p_1 = 0.6$, $p_2 = 0{\cdot}7$ and

$p_{12} = 0{\cdot}28$ in three distinct samples of a hundred balls each. After all, at least one of our samples may not have adequately represented the population. The law of large numbers indicates that the frequency in a sample approximates the proportion in the population with high probability, not with certainty. Note, however, that the violation of Boole's conditions due to a failure of randomness is a phenomenon of limited scope, for the probability that it will occur decreases as the size of the samples grow. Thus, if we have samples of extremely large sizes, and the violation of at least one relevant condition persists, we should look for an alternative explanation.

(b) *Measurement biases.* Even when the samples are perfectly random we can still observe a violation of Boole's conditions which is due to a bias or 'disturbance' introduced by our method of experimentation. Consider the following, somewhat artificial case. We take a random sample of college students and present them with the following question:

I. Indicate whether the following statement is true or false:

The present U.S. Secretary of State is Mr James Baker.

Suppose that the relative frequency of students who answer (correctly) 'true' is $p_1 = 0{\cdot}78$. Next we take a *second* sample of college students and ask them:

II. Indicate whether the following statement is true of false:

The present U.S. Secretary of State is Mr George Shultz.

The name sounds vaguely familiar (after all Mr Shultz was the previous Secretary of State). Add this to the fact that the natural tendency of people is to answer in the affirmative in matters of little personal consequence. The net result is that quite a few students will wrongly answer 'true' to question II, say $p_2 = 0{\cdot}33$.

Now take a third sample of students and present them with both question I and II (in that order). Whoever the present Secretary of State is, there is only one such person. Everyone knows that, so virtually nobody will answer 'true' to both questions. Thus $p_{12} = 0$ and $p_1 + p_2 - p_{12} > 1$. Note that the samples are random, at least there is no reason to believe they are not. Still the violation of Boole's condition is perfectly understandable, the reasons have been specified above.

This is a typical case where the results of an experiment depend heavily on the type of measurement and on the background against which it is performed. In such cases we can blame 'measurement biases' for the violation of Boole's conditions. In the artificial example described above, it is easy to correct the bias and remove the background 'interference'. But this may not necessarily always be the case. Sometimes measurement biases can be 'stubborn' and even unremovable in principle.

(c) *No distribution.* The law of large numbers asserts that, with high probability, the relative frequency of a property in a (finite) random sample approximates the proportion of that property in the population. For that reason we expect Boole's conditions to obtain even when the relative frequencies involved have been measured on distinct samples. Under normal circumstances, the existence of a population with a well-defined distribution of properties is an unproblematic assumption, which is even accessible to direct verification. In other circumstances this assumption becomes a matter of theoretical stipulation, which is empirically justified by the observable relative frequencies themselves.[14]

Whenever we are faced with statistical data, in the form of relative frequencies, we tend to attribute the results to some pre-existing distribution of properties in a hypothetical population. This attribution is just a special case of the human habit to look for causal explanations, even in cases when only the effects are present. But, as Hume [1739] taught us, the attribution of causal relations between two events which appear in temporal succession cannot be justified on *logical* grounds. If this skeptical thesis is valid, then skepticism with respect to stipulated causes is even more justified. Now, suppose that we observe a violation of Boole's conditions by relative frequencies measured on distinct samples. We may attribute this failure to one of our habitual (often implicit) assumptions, namely that there exists a well defined distribution of properties over some population, and the results of our measurements merely reflect this fact. Maybe there is no 'population', or, even if there is, there are no well-defined properties, existing independently of observation and distributed in a specific manner. All that exists are the phenomena themselves, which simply occur without cause.

(d) *Mathematical oddities.* It is logically possible to have a pre-existing distribution of properties, random samples and unbiased measurements and still obtain a violation of Boole's conditions. This can be achieved when the 'population' (or more precisely, the probability space) is the continuum. I shall briefly indicate how this can be done.[15] Consider the set of real numbers between zero and one, that is, the interval [0,1]. Let A be a subset of that interval. Suppose that we sample points from the interval at random. What then is the frequency of A-points (*i.e.* points belonging to the subset A) in the sample? For some subsets A, the so called Lebesgue measurable sets, there is a clear answer. The law of large numbers for this

[14] This point is best illustrated by the case of classical statistical mechanics. The thermodynamic averages are attributed to a particular distribution of molecular properties, the Maxwell–Boltzmann distribution. As is well known, the existence of the population (molecules) was hotly debated in the nineteenth century. Moreover, the particular choice of distribution could not have been justified on the basis of classical mechanics alone, but required, in addition, the assumption of equipartition (or statistical independence).

[15] More details can be found in Pitowsky ([1989b], pp. 147–75).

case asserts that, with high probability, the frequency of A-points is close in value to a definite number, called the Lebesgue measure of A. (In case A itself is an interval its Lebesgue measure is just its length.) If, however, A is not a Lebesgue measurable set, then the answer is no longer so sharp. In that case the frequency of A-points, in a large finite sample, can be any rational number between two extremes (called the inner and outer measures of A). That is, any rational number in that range, is an equally likely (or unlikely) candidate. Thus, we can even conceive of two sets A, B with the following properties: the relative frequency of A-points in one *random* sample is 1. The relative frequency of B-points in a second random sample is 1, while the relative frequency of $A \cap B$-points in a third random sample is zero! Needless to say, sets with such strange properties are very abstract creatures. The very existence of these non-measureable sets depends on the validity of the axiom of choice, an abstract set theoretical principle. This means that there is model of set theory (with the axiom of choice excluded) in which no non-measurable sets exist.[16] Since, however, the axiom of choice is relatively consistent, the 'construction' indicated above represents a logical possibility.

These are the cases, of which I am aware, where Boole's conditions might be violated. Another possibility, which has been neglected, is the case where we erroneously believe that some logical relation among the events obtains, and thus, wrongly expect some condition to be satisfied. Strictly speaking, this case does not represent a violation of Boole's conditions, but rather an error of judgement. It is quite irrelevant for our concerns. In all the examples we have discussed so far, the logical relations have been immediately recognized by common sense. In subsequent sections we shall consider microphysical events. The logical relations among these are also directly accessible to common sense and are, furthermore, given by theory.

4 QUANTUM VS. CLASSICAL PHENOMENA

We are now in a better position to attempt an answer to the WHAT question: *the difference between classical and quantum phenomena is that relative frequencies of microscopic events, which are measured on distinct samples, often systematically violate some of Boole's conditions of possible experience.*

A few words of clarification. The microscopic world reveals itself when events are registered in the (macroscopic) equipment of the laboratory. The apparatus can be a photographic plate, a particle counter, a bubble chamber, and so forth. By 'microscopic event' I mean the appearance of a black (macroscopic) dot on a photograph plate, the sound of a click in a counter, the

16 Solovay [1970]. The situation is in fact quite intricate; Solovay's model also requires a questionable axiom, the axiom of inaccessibility, see Shelah [1984].

appearance of a track in a bubble chamber, and the like. All events to be considered here occur at a more or less specific place at a specific time. Hence all measurements to be considered are essentially time-position measurements.[17]

Usually the appearance of such an event is attributed to the impact of a microscopic particle. But the term 'particle' is too loaded, for it implies a certain mental picture. I shall rather use the word 'thing'. Nothing which will be said subsequently depends on what these 'things' are and what shape and spatial location, if any, they have. It does not even matter whether 'things' exist at all.

When we observe a multitude of microscopic events we can measure relative frequencies. For example, we can count the number of events registered at a certain region in space and divide it by the total number of registered events. Among the events considered, certain logical relations may obtain. But when measured on distinct samples, the observed frequencies often violate one or more of Boole's conditions, dictated by those logical relations.

A natural question to ask is why should we measure the relative frequencies on distinct samples, and not on one and the same sample? The answer is very simple: in some cases we do not know how to perform simultaneous measurements. Microscopic events behave consistently in that respect. In case we know how to perform the relevant measurements simultaneously, then no violation of Boole's conditions is ever observed *even when the measurements are performed on distinct samples*. Conversely, if one or more of Boole's conditions is violated when distinct samples are concerned, then no method for simultaneous measurement is known to exist.

To illustrate my approach consider the paradigm case, the two slit experiment. We have a gadget which we call a source (a source of 'things'). In front of the source we put a screen with two slits and immediately behind it a photographic plate (Figure 2).

We may choose to perform any one of four experiments. (a) Close both slits, in which case no black dot appears on the plate, Figure 3a. (b) Open the upper slit and close the lower one. In this case black dots begin to appear on the plate and take the shape in Figure 3b. We can manipulate the source so the dots appear on the plate at a very slow rate, on the average one dot per 10 seconds, say. (c) Open the lower slit and close the upper slit to obtain the pattern in Figure 3c, which appears at the same slow rate. (d) Open both slits to obtain the pattern in Figure 3b, which appears at roughly twice the rate.

Now consider the dots which appear in the region A, indicated in Figure 3. If both slits are closed then no dots appear anywhere. Hence it is reasonable to assume that the dots which appear in A in experiment (b) occurred as a result of the fact that the upper slit has been opened. Thus some 'thing' ('particle',

[17] It can be argued that all measurements are ultimately time-position measurements. We shall not need this thesis there. Suffice to say that all experiments in which a violation of Boole's conditions occur have that nature.

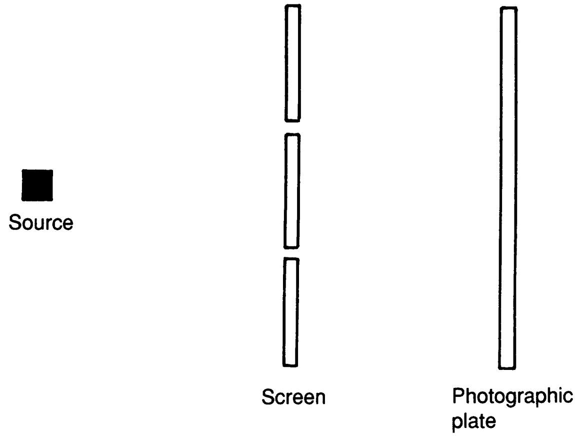

FIGURE 2

'wave', 'influence', 'vibration', or what have you) went through the slit. Likewise in experiment (c). Now consider experiment (d). By the same argument a dot which appears in region A is due to some 'thing' which went through the upper slit *or* the lower slit. *Note, this is not an exclusive 'or', the 'thing' may very well have 'travelled' through both slits at once.* The point is that there is no *other* possibility: we have verified that fact in experiment (a). Now, let p_1 be the relative frequency of dots appearing in region A in experiment (b), p_2 be the relative frequency of A-dots in experiment (c), and q be the relative frequency of A-dots in experiment (d). Boole's condition for 'or' dictates that $q \leqslant p_1 + p_2$. But in fact we count $q > p_1 + p_2$.

'Wait a minute,' you may say, 'isn't it obvious that an interference, a measurement bias, has occurred?' Well, it is not obvious at all! It is one possible explanation, which is far from being unproblematic (see Section **8**). It is true that physicists call this phenomenon 'interference', but the meaning of this term is precisely what is at stake.

In order to compare the above analysis with 'textbook' approaches, consider the following dialogue between a physics professor and her student:

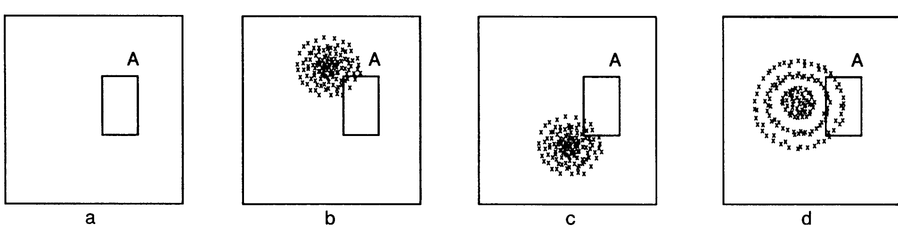

FIGURE 3

Teacher: This is the core of the quantum puzzle, the particle-wave duality. When one slit is opened we see the impact of scattered particles. The pattern is more or less the one we would expect from classical particles. When both slits are opened we would have expected picture (b) and (c) to be simply super imposed. Instead we get the interference pattern in (d). The matter waves, emerging from both holes interfere, and that's what we get.
Student: I do not quite understand this explanation, for two reasons. First, in all cases we see dots on the screen. These dots, if anything, appear as an impact of particles. This is true in pictures (b), (c), *and* (d). But this is really not my main concern. Second, very often in science we obtain results which are unexpected, which seem to defy common sense. This is true in classical physics too. I don't quite see why this particular experiment is any different, what is so revolutionary about it?
Teacher: Can't you see? We get an interference here.
Student: What do you mean by 'interference'?
Teacher: The pattern of dots in picture (d) is called 'interference'.
Student: O.K., but then again, what is so peculiar about it?
Teacher: It's the *number* of dots, you see? Here, in region A, we simply have far too many of them.

The discovery of puzzling microphysical phenomena, and the development of quantum theory, occurred at roughly the same time. It is thus only natural that the language of q.m. is being used to depict the phenomena. 'Matter waves', 'state functions', 'collapse', 'commutation relations', and 'interference' are highly theoretical terms. If we are to understand the revolutionary character of q.m. we should try as much as possible to avoid those terms when we describe the phenomena themselves. For otherwise we simply beg the question. Indeed, the puzzling aspects of microphysics reside in the phenomena themselves, and not just in their theoretical explanations.

5 QUANTUM THEORY

How does the formalism of q.m. incorporate these facts? One of the major purposes of q.m. is to organize and predict the relative frequencies of events observable in various experiments—in particular the cases where Boole's conditions are violated. For that purpose a mathematical formalism has been invented which is essentially a new kind of probability theory. It uses no concept of 'population' but rather a primitive concept of 'event' or more generally 'observable' (which is the equivalent of the classical 'random variable'). In addition, to every particular physical system (which can be one 'thing'—an electron, for example—or consists of a few 'things') the theory assigns a state. The state determines the probabilities for the events or, more generally, the expectations of the observables. What this means operationally is that if we have a source of physical systems, all in the same state, then the relative frequency of a given event will approach the value of the probability, which is theoretically determined by the state.

For certain families of events the theory stipulates that they are commeasurable. This means that, in every state, the relative frequencies of all these events can be measured on one single sample. For such families of events, the rules of classical probability—Boole's conditions in particular—are valid. Other families of events are not commeasurable, so their frequencies must be measured in more than one sample. The events in such families nevertheless exhibit logical relations (given, usually, in terms of albebraic relations among observables). But for some states, the probabilities assigned to the events violate one or more of Boole's conditions associated with those logical relations.[18]

It is also important to note what quantum theory *does not* say. It does not stipulate any physical cause for the violation of Boole's conditions, it only predicts that it will occur. In particular q.m. does not provide any *dynamic* mechanism of measurement biases which might have explained the violation of Boole's conditions. This is the notorious 'collapse of the wave packet' or 'measurement problem'. These expressions are used by physicists to indicate that the results of measurements are not determined, not even probabilistically determined, by any dynamic theory of measurements. Measurements, in other words, are not depicted in q.m. as processes that take a finite positive amount of time.

It goes without saying that q.m. does much more than indicated above. Most notably, it incorporates physical symmetries in a much more profound way than classical theories do. Very often, the symmetries form the basis for the prediction of the probabilities.

6 ON THE EDGE OF A LOGICAL CONTRADICTION

Why is it that relative frequencies of microscopic events behave in the peculiar way they do? Similar questions asked in different contexts seem to be devoid of *physical* content. Why is it that the velocity of light is constant in every inertial reference frame? It is not clear what kind of answer can qualify as a reasonable one, nor is it clear that the question is at all meaningful. The principle of relativity—as with many other physical principles—is taken as a basic assumption which requires no further deliberation. There always comes a point where the question 'why are things the way they are?' stops being a question in physics. 'That's the way God made it, that's the way He wants it to be'; what else can we say?

[18] It is interesting to determine which of Boole's conditions can be (theoretically at least) violated, and what then is the extent of violation. It turns out that all but very few of the simplest conditions are vulnerable. See Pitowsky ([1989b], pp. 63–76).

Perhaps this is also the point to stop asking questions about quantum phenomena. We have a theory which accurately predicts a large variety of empirical facts. Some of these facts do not conform to our traditional analytic conception of probability. This state of affairs should be taken as a fact of life, which is perhaps not entailed from any deeper law of nature.

From an empiricist point of view there is much to be said in favour of this minimalist position. The empiricist, who suspects scientific explanations in general,[19] has a good case here. For years, scientists have attempted to derive the predictions of quantum mechanics from principles which appeared to them to be more basic and intuitive. These theories were often *ad hoc*, and in any case never resulted in a genuine empirical extension. At best, these theories reproduced the predictions of q.m., never more, and in a much more cumbersome fashion. As long as this is the case, why should we go further?

A more moderate empiricist may accept the WHY question as legitimate for instrumental reasons. In spite of the continuous failure to 'solve' the quantum puzzle, the effort may be worthwhile. In the future it may result in some genuinely new physics. At any rate, the dispute between the moderate and extreme empiricists is about empirical judgements, whether the WHY question will turn up to be a fruitful *scientific* research problem.

I believe, that in the case of q.m., there is a philosophical interest in the WHY question, quite independently of its scientific merits. Physics is not an isolated enterprise, detached from other fields of knowledge. If epistemology is to become a unified coherent pursuit, physical facts should be made consistent with the rest of our beliefs. Here, and I think only here, lies the *philosophical* importance of the quantum revolution.

A violation of Boole's conditions of possible experience cannot be encountered when all the frequencies concerned have been measured on a single sample. Such a violation simply entails a logical contradiction; 'observing' it would be like 'observing' a round square. We expect Boole's conditions to hold even when the frequencies are measured on distinct large random samples. But they are systematically violated, and there is no easy way out (see below). We thus live 'on the edge of a logical contradiction'. An interpretation of q.m., an attempt to answer the WHY question, is thus an effort to save logic.[20] Not to 'save the phenomena', as the moderate empiricist might believe, but to save logic. We simply attempt to incorporate microphysical phenomena to the rest of our knowledge, to form a coherent picture. Physics may very well get along without such an effort, science without philosophy may not be so blind after

[19] This empiricist position is most forcefully argued in van Fraassen [1980].

[20] Some may even go as far as giving up logic, as Birkhoff and von Neumann [1936] did. Although the *motivation* behind quantum logic can be clearly seen from the above analysis, I still think it is an indefensible position. It encounters physical as well as philosophical problems. See Pitowsky [1989b], Chapter 4.

all. But the position of physics *vis à vis* epistemology will then remain on a shaky ground.

In this respect q.m. is different from relativity. The latter posed a threat to some basic *physical* intuitions, notably those concerning space and time. Such a threat is more easily contained within physics itself. Quantum mechanics, by contrast, *appears* to question intuitions whose range of validity lie beyond physics, in fact beyond any one specific domain of inquiry. The interpretation of q.m. is supposed to provide an explanation as to why this is only an appearance, why there really is no threat to our logical conception, only an illusion of one. There are various of such possible interpretations, each with its own weaknesses. They more or less follow the cases compiled in Section 3 and I shall discuss them in turn. I do not by that pretend to exhaust all the possibilities suggested in the literature, only those which seem to me to be reasonable, at least to some extent.

7 PRISM MODELS, THE FAILURE OF RANDOMNESS

The violation of Boole's conditions by the relative frequencies of microphysical events persists and stabilizes over very large samples. Moreover, each sample can be tested for its intrinsic randomness and in that respect the samples are as random as one can get. Therefore, it seems quite unreasonable to attribute the violation of Boole's conditions to a failure of randomness. This would seem like a grand conspiracy of Nature. Conspiracy, that is, against human scientists.

Indeed, there is one version of the 'failure of randomness' approach, which is outlandish. According to that version, the particular experimental set-up 'conspires' with the *source* to choose a non-representative sample so as to accommodate the predictions of q.m. One may go even further and claim that the will of the scientist, to perform this or that experiment, somehow selects the appropriate ensemble. Ideas like that are not just unreasonable, they are empty in the sense that they lack any explanatory (let alone predictive) power.

A more reasonable approach in that direction is represented by the prism models due to Fine [1982b]. A typical experimental set-up in microphysics consists of three parts. There is a source from which 'things' emerge, a testing device in which 'things' are manipulated, and finally a detector in which the results are recorded. (In the two slit experiment these parts are the source, screen, and photographic plate respectively.) Fine notes:

> Every real experiment loses track of some of its population. . . . Some wander off after reaching the mansion, some get lost in their wing, some will stubbornly refuse to respond to certain tests or will respond in an unintelligible way, some responses will be lost before they are recorded—or recorded incorrectly or illegibly.

In other words, in any real experiment some of the original population fails to be detected for some reason or other. In many cases, in particular in some crucial experiments (such as the Einstein–Podolsky–Rosen experiment, see next section), it is known that the detectors themselves have low efficiency. The results recorded in all these experiments conform to the predictions of q.m. *provided* that we assume that the failure to detect is random, that is, independent of the test performed between source and detector. This is a very reasonable assumption indeed.

If, however, we assume that the percentage of undetected incidents is correlated with the state of the testing device, we can arrange the numbers in such a way so as to conform to Boole's conditions.[21]

Consider, for example, the two slit experiment. Many of the 'things' that emerge from the source get lost on the way to the photographic plate, and thus do not leave black dots on it. If we assume that the percentage of undetected incidents depends, in a complex way, on whether one or two slits are opened, we can 'explain away' the interference effect. In other words, if we can somehow increase detection efficiency, the interference will disappear.

The prism models are thus also based on a conspiracy—this time a conspiracy between the experimental set-up and the *detector*. Their advantage is that they are testable in principle. More efficient detectors should lead to weaker interference, that is, smaller violation of Boole's conditions. I think it is fair to say that no one really believes this will happen.

8 MEASUREMENT BIASES: THE FAILURE OF LOCALITY

The most straightforward way to explain the deviations from Boole's constraints is to attribute them to measurement biases. As noted in the third section, the specific design of an experiment and the conditions present at the time of measurement may shift the 'real' value of a parameter, with the result that one or more of Boole's conditions is violated. Moreover, it may be the case that such a bias is built into our equipment and cannot be removed by improved technology. Indeed q.m., at least its non-relativistic version, can be consistently interpreted in this manner.

Theories which incorporate measurement biases, and explain the statistical outcomes of microphysical experiments by reference to a dynamic picture of the measurement processes, are usually called 'hidden variable theories'. The reason is that such theories include physical parameters, which are not defined in q.m. itself, and whose dynamic changes during the measurement processes are responsible for the statistical peculiarities. The most elaborate theory of that kind is due to Bohm [1952]. This is a deterministic theory, which assumes only the principles of Newtonian physics. It is thus a theory of particles, where each particle has a definite position and momentum at all times. In addition to

[21] It is important to note that one can construct local prism models for the Einstein–Podolsky Rosen experiment (see next section). Fine [1982c].

the usual forces of nature, the particles are also influenced by a quantum force. The effect of this force is to shift the trajectory of the particles, so that their final destination conforms to the one encountered in the laboratory. The quantum force, or the quantum potential from which it is derived, is extremely sensitive to very slight changes in the boundary conditions. For example, the opening of the second slit, in the double slit experiment, changes the spatial form of the quantum potential dramatically, with the result that particles move in trajectories utterly different from those encountered in the single slit case.[22]

This extreme sensitivity of the quantum potential is also responsible for the uncertainty relations. Particles possess definite position and momentum at each moment but we are unable to determine their values simultaneously. The reason is that the conditions present during a momentum measurement change the value of the quantum potential which have existed prior to the action of measurement, with the result that the position is shifting. The statistical average of such biases agree with the uncertainty relations. In fact, all the predictions of Bohm's theory agree with (non-relativistic) q.m.

I agree with the view expressed by Bell [1987] that Bohm's theory represents an important alternative interpretation of q.m. and that it is regrettable that it is not routinely taught in physics courses.

If everything is so perfectly nice, you may ask, why should we go further? But Bohm's theory, in fact all theories which incorporate measurement biases in an attempt to solve the quantum puzzle, suffer from a serious *physical* disadvantage. They seem to be incompatible with special relativity, at least with the spirit of this theory. It turns out that in order to explain the results of some experiments, we have to assume that a measurement performed in Jerusalem induces an instantaneous significant bias in a simultaneous measurement performed in New York (in the case of Bohm's theory, this means that the quantum potential changes its value instantaneously all over space, in utter contradiction to the relativity principle).

This amazing fact was discovered by Bell [1964] and a detailed version of the argument is reproduced in Appendix 3. In this case, again, we face a violation of some of Boole's conditions of possible experience, associated with the events which occur in the Einstein–Podolsky–Rosen (E.P.R.) experiment. Boole's conditions for that particular case are called Bell inequalities (or Clauser–Horne inequalities in a slightly different version, see Appendix 2). At least one of these inequalities is violated by the observed frequencies. If we attempt to attribute this effect to a measurement bias, we must assume that the bias takes a very peculiar form: it propagates instantaneously across space, and remains at the same significant level at all distances.

Bell's remarkable discovery casts great doubt on the idea of measurement biases, for the following reasons:

22 A graphic representation of quantum potentials and the associated particle trajectories in various situations is provided in Dewdney and Hilley [1982].

(a) As noted before, all theories which incorporate measurement biases, Bohm's theory included, do not predict anything which is not already predicted by q.m.
(b) There is no independent evidence that systematic unremovable measurement biases exist in *any* experiment, E.P.R. in particular. The only evidence for a bias is the violation of Boole's condition itself. Thus, for example, in the E.P.R. case, we cannot manipulate the 'bias' to achieve faster-than-light signalling.
(c) In spite of this fact, the *idea* that a real measurement bias occurs is incompatible with relativity. In Bohm's theory, in particular, this means that the quantum potential is not Lorentz-invariant, even though the probabilities (statistical averages) are. Hence the measurement bias idea reduces special relativity to the status of a statistical rather than a principal theory of space and time.

The conjunction of these facts means that it is doubtful whether measurement biases exist in the first place. It does not preclude the possibility altogether—I do not believe that *any* argument can eliminate the idea completely—but it casts great doubt on the hidden variables interpretation.

The situation is somewhat analogous to the case of the ether. It is often maintained that Einstein's special relativity eliminated the ether from physics. This is true in the sense that very few people still believe the idea. But from a logical point of view, the old theory due to Lorentz, which attributes the relativistic effects to the strange dynamic properties of the ether, is not *necessarily* false. It is simply highly implausible. It is really Ockham's razor at work here; why should we assume the existence of strange dynamic effects, when everything follows from a simple consistent kinematic picture?

By analogy, q.m. assumes no dynamic picture of measurement processes and biases, and still gets the correct results. All attempts to incorporate a dynamic picture take a highly undesirable theoretical form, while yielding no new predictions. It seems to be just the occasion to apply Ockham's razor once more.

Yet the WHY question still persists, for the reasons indicated in Section 6. We must therefore consider other, more radical, possibilities.

9 NO DISTRIBUTION: THE FAILURE OF CAUSALITY

When we are faced with statistical data, in the form of frequencies, averages, cross-sections and the like, we automatically tend to interpret the results in terms of a distribution of properties in some hypothetical population. As already noted, this tendency is a special case of the human habit to look for causal explanations, even in cases when only the effects are present. Given the violation of Boole's conditions, and the problematic character of the hidden variables approach, we may consider the possibility that the hypothetical distribution simply does not exist.[23] All that exist are the phenomena themselves and the correlations among them.

This brings us to the tricky question of realism. It is often maintained that quantum mechanics forces us to choose between 'locality' and 'realism'. Apart from the fact that, logically speaking, this is not a case of an excluded middle (see next section), the term 'realism' is used here in a restricted and, philosophically speaking, quite peculiar sense. When we deny that *some* properties exist (independently of observation), and are distributed in a certain way, we do not necessarily reject 'realism' as a metaphysical position.[24] After all, the peculiarities of quantum statistics are associated with certain observables—but not all. There are observables, such as electric charge, rest mass, baryon number, and the like, which are not 'complementary' to any other observable, and thus are not involved in difficulties of the kind discussed above. These observables allow us to define the identity of particles in the first place. There are no *logically* compelling reasons to take an antirealistic position with respect to those (though there may exist philosophical reasons to argue against physical realism in general, but this is not my concern here). In other words, the assumption that particles (or 'things') exist together with *some* of their properties is compatible with experience, and no further assumption (such as the existence of measurement biases) is required to make it consistent. For this reason, I believe that 'realism' is not the issue here.

What is at stake is the idea of causality. The 'no distribution' approach takes the view that certain phenomena, or more precisely, *certain aspects of certain phenomena*, have no causal explanation. They simply occur and that is it. There is no 'deeper reality' which causes them to occur; the phenomena themselves are their deepest explanation.

There are some similarities between this view and the empiricist view discussed in Section 6, but there are also important differences. The empiricists suspect the idea of 'scientific explanations' and causal explanations in particular. In their view, the function of theories is to save the phenomena, that is, to organize data and predict (via logical deduction) new phenomena. The idea that theories *really* provide us with causal explanations is an illusion (even in the case of classical science).

The view discussed here is more limited in scope. One may take a less skeptical position with respect to scientific explanations in general, and still deny the existence of causes in some special cases. One may argue that as far as certain aspects of microphysical phenomena are concerned, the denial of casual explanations is forced upon one, or is at least compelling. This is not a matter of empiricist ideology but rather a matter of contingency.

[23] An argument to that effect which is based on the E.P.R. correlations is in van Frassen [1982].
[24] This point is argued in detail in Ben-Menachem [1988].

10 MATHEMATICAL ODDITIES REVISITED

For whatever its worth, we can provide an 'explanation' of microphysical peculiarities along the lines discussed at end of Section 3.[25] This means that it is logically consistent to maintain both causality and at the same time to deny the existence of measurement biases. The price, however, is quite heavy; we apply abstruse mathematics to prove the existence of certain non-measurable 'distributions', whose only function is to 'solve' a logical puzzle. Ockham's razor can be equally (or even more fiercely) applied to this case.

11 CONCLUSION

Human beings are often incapable of distinguishing truth from falsity. Recognizing that a given statement, say a hypothetical 'law of nature', is true requires a special relation between humans and the external world, a relation which apparently does not exist. Identifying the truth value of certain mathematical statements (*e.g.* the continuum hypothesis) requires an even more mysterious relation, that of human beings to the realm of ideas, which, again, seems not to exist. But recognizing consistency or, more precisely, an inconsistency, a paradox requires no relation to anything external, only the internal powers of computation, which we do seem to possess to a certain extent. This observation is the core of the formalist approach, due essentially to Hilbert [1918]: consistency, or rather the avoidance of a contradiction, is ultimately the only safeguard in the pursuit of knowledge.[26] We would have liked to do better for sure, but we have very little choice in the matter.

The presence of a paradox and the attempt to avoid it have been the themes of this paper. I think that it is significant that the paradoxical aspects of microphysics are directly associated with the historical origins of modern logic—the work of George Boole. We are close to witnessing what, in Boole's terms, amounts to 'an impossible experience'. Of course we cannot quite witness a logical contradiction with our own eyes, but this is as close as we can get. There are no easy ways out, all explanations seem ultimately like excuses; at least this is how I feel.

Perhaps this is so because we do not understand completely the nature of the phenomena involved, and future developments in science will lead us to the answer. This may very well be the case, but judging from the experience of the past seventy-five years, I suspect that it is not. Perhaps it is all a psychological delusion, we simply have to get used to the facts, and the

[25] A detailed model for the E.P.R. case, constructed along similar lines, is in Pitowsky [1983].

[26] Hilbert erroneously thought that the power of computation is sufficient to recognize consistency. He was right, however, with respect to *inconsistency*. Note that Popper's falsificationist strategy is a special case of the formalist approach. We cannot recognize a theory as true (the problem of induction), but we can attempt to falsify it. For all falsification requires is the ability to recognize a contradiction (between a theory and a particular observation).

questions will somehow disappear. All scientific revolutions are hard to grasp at the outset. But I do not believe that this is the case either. We are not at the beginning of a process in its pre-analytic stage. If anything, the discoveries of the last thirty years have deepened the paradox, and caused more and more scientists and philosophers to raise the question. In that respect q.m. is the greatest scientific revolution of all, for there is still something about it which we do not understand, and perhaps never will.

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APPENDIX I AN ALGORITHM FOR BOOLE'S CONDITIONS

We shall consider first a special case. We are given the probabilities $p_1, p_2, \ldots, p_n$ of n events $E_1, E_2, \ldots, E_n$, and the probabilities p_{ij} of some, not necessarily all, their pair conjunctions $E_i \cap E_j$, where $\{i,j\}$ range over some set of pairs S.

In order to drive Boole's conditions for $p_1, \ldots, p_n \ldots p_{ij} \ldots$, we take n propositions $a_1, \ldots, a_n$ and consider the truth table for these propositions and the joints a_i & a_j where $\{i,j\} \in S$. A typical row in the truth table has the form:

a_1	a_2	...	a_n	...	a_i & a_j	...
ε_1	ε_2	...	ε_n	...	$\varepsilon_i\varepsilon_j$	...

where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are either zero or one. There are 2^n such rows each with $n+|S|$ entries, where $|S|$ is the cardinality of S. Consider each row as a vector in an $n+|S|$ dimensional real space. The convex hull of the 2^n vectors is a (full dimensional) polytope, called a correlation polytope, and denoted by $c(n,S)$. The facet inequalities of $c(n,S)$ are just Boole's conditions of possible experience for n events and their joints in the set S. This is the case because the following theorem obtains:[27]

Theorem 1: let $(p_1, \ldots, p_n, \ldots, p_{ij} \ldots)$ *be an* $n+|S|$ *dimensional real vector. This vector is an element of* $c(n,S)$ *if and only if there exists some probability space* (X,Σ,μ) *and some events* $E_1, \ldots, E_n \in \Sigma$ *such that* $p_i = \mu(E_i), 1 \leqslant i \leqslant n$ *and* $p_{ij} = \mu(E_i \cap E_j)$, $\{i,j\} \in S$.

In order to determine Boole's conditions for this case we have to determine the facets. First we note that since all vertices of $c(n,S)$ are zero-one vectors the coefficients of the facet inequalities are bounded. The bound (which depends on n) can be quite easily estimated. Next we guess $n+|S|+1$ positive or negative natural numbers, within this bound: $b_0, b_1, \ldots, b_n, \ldots b_{ij}, \ldots$ Subsequently we check whether the inequality

[27] For a proof see Pitowsky ([1989b], pp. 22–3).

$$b_0 + \sum_{i=1}^{n} b_i\varepsilon_i + \sum_{\{ij\}\in S} b_{ij}\varepsilon_i\varepsilon_j \geqslant 0$$

is satisfied for all zero-one sequences $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$. If it is, we finally establish the affine dimensionality of the space of all $n+|S|$ dimensional vectors $(\varepsilon_1, \ldots, \varepsilon_n, \ldots, \varepsilon_i\varepsilon_j \ldots)$ for which *equality* holds. If it is maximal then

$$b_0 + \sum_{i=1}^{n} b_i p_i + \sum_{\{ij\}\in S} b_{ij} p_{ij} \geqslant 0$$

is one of Boole's conditions.

This is a cumbersome and highly inefficient procedure. Unfortunately no better method is known to exist, and it is highly probable that none exists. The determination of Boole's conditions for this case is directly related to the most important open problems in the theory of computational complexity (Pitowsky [1991]).

The general case follows the same pattern. Suppose that we are given the probabilities $q_1, \ldots, q_k$ of k events $A_1, \ldots, A_k$ where each A_j is a Boolean function of events $E_1, \ldots, E_n$: $A_j = \varphi_j(E_1, \ldots, E_n)$. This means that A_j is given in terms of a sequence of operations among intersection, union, and complementation on $E_1, E_2, \ldots E_n$. We consider the truth table for the propositional formulas $\varphi_j(a_1, \ldots, a_n)$ where intersection means conjunction, union stands for disjunction and complementation is negation. Each row in the table has k entries, for the k propositions, and there are 2^n rows. The convex hull of the rows in the k dimensional space is a polytope. The inequalities and equalities which describe it are Boole's conditions for that case. This is so because theorem 1 can be easily generalized to cover all possible logical relations.

APPENDIX 2 CLAUSER–HORNE INEQUALITIES

These are Boole's conditions in the following case: we are given the probabilities p_1, p_2, p_3, p_4 of four events E_1, E_2, E_3, E_4 and the probabilities $p_{13}, p_{23}, p_{14}, p_{24}$ of four (out of the six possible) pair joints, namely $E_1 \cap E_3$, $E_2 \cap E_3$, $E_1 \cap E_4$, $E_2 \cap E_4$. Again, we consider the truth table:

a_1	a_2	a_3	a_4	a_1 & a_3	a_1 & a_4	a_2 & a_3	a_2 & a_4
ε_1	ε_2	ε_3	ε_4	$\varepsilon_1\varepsilon_3$	$\varepsilon_1\varepsilon_4$	$\varepsilon_2\varepsilon_3$	$\varepsilon_2\varepsilon_4$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ are either zero or one. There are sixteen rows in the table each with eight entries. The facet inequalities for the polytope which results are:[28]

$$p_{ij} \geqslant 0, \quad p_i \geqslant p_{ij}, \quad p_j \geqslant p_{ij}, \quad p_i + p_j - p_{ij} \leqslant 1 \quad i = 1,2 \quad j = 3,4 \quad (7)$$

[28] Sufficiency was first proved by Fine [1982a]. See also Pitowsky ([1989b], pp. 27–30).

and

$$\begin{aligned} -1 &\leqslant p_{13}+p_{14}+p_{24}-p_{23}-p_1-p_4 \leqslant 0 \\ -1 &\leqslant p_{23}+p_{24}+p_{24}-p_{13}-p_2-p_4 \leqslant 0 \\ -1 &\leqslant p_{14}+p_{13}+p_{23}-p_{24}-p_1-p_3 \leqslant 0 \\ -1 &\leqslant p_{24}+p_{23}+p_{13}-p_{14}-p_2-p_3 \leqslant 0 \end{aligned} \quad (8)$$

The last eight inequalities are Clauser–Horne inequalities.

APPENDIX 3 THE E.P.R. EXPERIMENT

This experiment has been described in so many publications that I shall not repeat the details, only the aspects which are revelant in the present context. As usual we consider a source of electron pairs (or 'things') whose presence can be detected by apparrata located on both sides of the source, 'left' and 'right'. We can choose to perform a polarization experiment, by a Stern Gerlach (S.G.) magnet, on one of the electrons, in a given direction, or polarize them both in identical, or distinct directions. Let x, y, z, w be four directions in physical space. The following table provides the results of eight such possible experiments. (Needless to say, the frequencies are measured on distinct samples.)

Event	Experiment	Relative frequency
E_1-left electron up x-direction	S.G. magnet x direction left, no magnet on right	$p_1=\frac{1}{2}$
E_2-left electron up y-direction	S.G. magnet y left no magnet on right	$p_2=\frac{1}{2}$
E_3-right electron up z-direction	no magnet on left S.G. magnet z right	$p_3=\frac{1}{2}$
E_4-right electron up w-direction	no magnet on left S.G. magnet w right	$p_4=\frac{1}{2}$
$E_1 \cap E_3$-left electron up x *and* right electron up z	S.G. magnet x left S.G. magnet z right	$p_{13}=\frac{1}{2}\sin^2(\frac{\widehat{xz}}{2})$
$E_1 \cap E_4$-left electron up x *and* right electron up w	S.G. magnet x left S.G. magnet w right	$p_{14}=\frac{1}{2}\sin^2(\frac{\widehat{xw}}{2})$

$E_2 \cap E_3$-left electron up y *and* right electron up z	S.G. magnet y left S.G. magnet z right	$p_{23} = \frac{1}{2}\sin^2\left(\frac{\widehat{yz}}{2}\right)$
$E_2 \cap E_4$-left electron up y *and* right electron up w	S.G. magnet y left S.G. magnet w right	$p_{24} = \frac{1}{2}\sin^2\left(\frac{\widehat{yw}}{2}\right)$

For some choices of directions x, y, z, w, the relative frequencies violate one of Boole's conditions for that case namely, a Clauser–Horne inequality (8). For example, if we choose x, y, w, to be coplanar and 120° apart, and $y = z$ we have:

$$p_{13} + p_{14} + p_{24} - p_{23} - p_1 - p_4 = \tfrac{3}{8} + \tfrac{3}{8} + \tfrac{3}{8} - 0 - \tfrac{1}{2} - \tfrac{1}{2} = \tfrac{1}{8} > 0.$$

In itself, this is just another case of a violation of Boole's conditions by quantum frequencies. We have seen that this occurs often, and in a diversity of experimental situations. If, however, we attempt to explain the outcome by reference to 'measurement biases', the result is quite peculiar. I shall not attempt any formal proof here, but proceed along an informal line of reasoning.

First note that the distance between the magnets (or magnets and source) can be made very large while the violation of Boole's condition remains at the same constant level, independent of the distance. Next, note that the relative frequencies of the events measured on one side (the first four events) is constant $\frac{1}{2}$, and is independent of the direction. Now, the frequency of *at least one* of the joint events, say $E_1 \cap E_3$, is simply too large to conform to Boole's conditions. Suppose that this has occurred because of a measurement bias. The only kind of bias that one can think of is the following: the presence of the magnet in the z-direction on the right had caused some of the *left* 'things', that would have otherwise gone down, to go up in the x-direction.

To illustrate this point more clearly suppose that in the fifth experiment, $E_1 \cap E_3$, we use a sample of 160 pairs. Suppose moreover that this is the only experiment among the eight in which a measurement bias occurs. (This is not a realistic assumption but any other assumption leads to identical conclusions.) Hence, if no measurement bias had occurred, and Boole's conditions had been satisfied we would have had measured a frequency $q_{13} \leqslant \frac{1}{4}$ for the event $E_1 \cap E_3$. *In fact* we get $p_{13} = \frac{3}{8}$. This means that our experiment had wrongly reported the state of at least 20 pairs. But the marginals, the probabilities of E_1 alone, and E_3 alone, remain constant $\frac{1}{2}$. How can this be?

We can easily think of an answer. The distribution *without bias* is e.g.:

event:	$E_1 \cap E_2$	$\bar{E}_1 \cap E_3$	$E_1 \cap \bar{E}_3$	$\bar{E}_1 \cap \bar{E}_3$	E_1	E_3
probability:	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$

while the bias causes us to observe:

event:	$E_1 \cap E_3$	$\bar{E} \cap E_3$	$E_1 \cap \bar{E}_3$	$\bar{E}_1 \cap \bar{E}_3$	E_1	E_3
relative frequency:	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$

Hence the presence of the S.G. magnet in the z-direction on the right had caused twenty of the left 'things', that would have otherwise gone down, to go up in the x-direction.

But the distance between the magnets is arbitrarily large. We can introduce the magnet instantly and thus induce an instantaneous change in the physical state of some 'thing' far away. Of course we do not know that this is really the case, because the marginals remain constant $\frac{1}{2}$. Therefore, we cannot transmit messages in this way, nor can we directly verify that a bias really occurs.

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