Notes for *Symmetry and the Beautiful Universe*,
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Developed incrementally in order of importance, as time permits.

This color indicates pieces of definitions of symmetry and related terms. I need to build this as I go along.

Ch. 1-5

p. 14-5: Working definition of symmetry

p. 39-43: Time of supernova that created the solar system. Natural uranium reactors at Oklo show that laws of physics have not changed in last 3 billion years. Mentioned a few more times in book.

p. 66-9: math, physics, and models. Some history.

p. 73: intro to Noether's theorem.

p. 93: somewhat different definition of global vs. local.

Chapter 6, Inertia

p. 118: Symmetry of uniform states of motion. Laws of physics don't change for different values of uniform velocity.

p. 130: A symmetry is an equivalence of things. Equivalence of all states of uniform motion for the laws of physics is almost a symmetry of nature. Galilean invariance/relativity/symmetry.

Diffuse: NT is a general proof that a continuous symmetry implies a conserved quantity and vice versa. Therefore, by NT, symmetry of translation in space implies conservation of momentum. It is mentioned that this can also be shown fairly simply just by manipulating the equations, without reference to NT. This can be seen as either a specific example of NT, or a proof of it for the specific case of momentum conservation.

Chapter 7, Relativity

The important things in this chapter:

- Galilean relativity and Einsteinian (special) relativity are almost the same. They both claim that all the laws of physics are the same, regardless of speed and direction of uniform motion.
- Galileo assumed that everyone would see the same *elapsed time between events*, regardless of their motion. In other words, universal time is a law physics.

- For various reasons, Einstein saw that this is wrong. (It was shown experimentally to be false by the Michaelson-Morley experiment.) Einstein's alternative assumption was that everyone will see the same *speed of light*, regardless of their speed relative to its source.

- The selection of invariant speed of light instead of invariant elapsed time to be considered a law of physics is a *hypothesis* that must be verified experimentally. Einstein won, at least so far.

- All the other surprising things about special relativity come directly from this small difference.

p.150: Galilean relativity – laws of physics are believed to be invariant under differences of uniform motion. (Alt wording: differences of uniform motion can be described as translation in the velocity part of parameter space. Does this matter?)

p. 151-2: Galilean relativity expects and depends on absoluteness of time intervals. The validity of Galilean relativity as a symmetry of nature is something to determine experimentally (or refute logically from other experimental evidence, as Einstein did.) It's wrong, since no velocity may exceed that of light, and speed of light is found to be independent of various states of uniform motion.

p. 153-4: Einstein changed the description of the laws under boosts (changes of uniform motion) by assuming that the speed of light didn't change and could not be exceeded. This means that time intervals (Δt) are NOT invariant, but that $c^2(\Delta t)^2-(\Delta x)^2$ ("the interval") is.

p. 160: If symmetry is the controlling principle, as Einstein believed, then all laws must be invariant under the relativistic boost transformation, and all laws must approach the Newtonian form as v -> 0 (since those laws seem to be valid there). This is a strong constraint on the form of possible laws.

p. 162: energy relations of special relativity.

Chapter 8, Reflections

The real meat of the book starts in this chapter. It is about reflection symmetry. There are many kinds of reflections. The chapter starts out easy with mirror reflection. It turns out in 3-space that is the same as moving all x values to –x (or along any other axis you might want). That's called parity inversion (P). You can do the same thing with time. But, since time flows in a particular direction whether we like it or not, this feels different to us. It looks much different to see a movie run backwards, but, other than that, it's really very similar to reflection along a spatial direction. This is called time reversal (T). What other physical properties might be susceptible to some kind of reflection? What has only two possibilities that might be reversed? How about electric charge? It's either + or -. What happens if you reverse them? It turns out this is the difference between matter and antimatter. This is called charge conjugation (C).

Notice two things about the previous paragraph: First, there is a terminology problem. The concept of reflection has acquired three new names: inversion, reversal, and conjugation. Don't let this obscure the common idea. Second, nothing was said about the laws of physics. Whether any of the C, P, or T reflections changes the laws of physics is an important question, and must ultimately be answered by experiment. For P, it superficially looks like the laws don't change at all. For T, it looks like time reversal makes a mess of the laws of physics. But if you think about behavior of atoms on a microscopic scale, then T doesn't seem to make any difference either. For C, that doesn't seem to change the laws either. Each of these kinds of reflection appears to leave the laws of physics unchanged.

Chapter 8 is about defining these three reflections and exploring what effect they have on the laws of physics. Superficially, they all look like nothing changes. More detailed experiments show that each reflection does in fact change something about how the laws work. In all three cases, the difference involves the weak interaction. Importantly, there is a theorem that if you simultaneously reverse C, P, and T, the laws are NOT changed. However, nature is not obliged to respect theorems, so even this is a matter to be determined by experiment.

p. 166: Reflection in a mirror is a discrete binary symmetry. You can't do it part way (discrete), and if you do it twice, you get back where you started (binary). If you only do it once, does that affect the way the laws of physics work?

Noether's theorem (NT) only applies to continuous symmetries. However, discrete symmetries can also be associated with conservation laws. These can be very important in quantum theory.

p. 168-9: Consider some physical object. What happens if you reflect it in space? If we think of it as "symmetrical", it will be unchanged. If it is "asymmetrical", it will be different, but if you reflect it again, it changes back. The first is called a "singlet", because there is only one state it can be in with regard to this transformation (reflection). The other is a "doublet", because under reflection, it has two possible states. "Invariant under reflection" means the first case, a singlet. Due to the inherently binary nature of reflection, these are the only two possibilities. (Actually, this and other symmetries can be partial. Consider the human body. It is invariant under reflection if you only look at the outside. On the inside, the organs are arranged unequally, so if you are interested in that, the body is a doublet. Often it is necessary to identify which properties you expect to be preserved, and which you do not.)

p.172-3: This situation is of interest to physics because the "object" being reflected can be any physical situation and the laws that describe it. The reason for the interest is to see if <u>all</u> situations behave the same when reversed, or not. If so, then reflection symmetry (P, parity symmetry) is a "symmetry of nature".

p. 173-81: In most experiments it appears that everything (i.e. the laws of physics) is invariant under parity inversion, or spatial reflection. How can we be sure? Do more experiments. P. 176-81 describe early experiments that show that the laws of the weak interaction <u>are</u> different when reflected.

p. 181-4: Time reversal is another similar type of reflection. Most superficially, this changes everything, and is nowhere near a symmetry. But when we look on the scale of individual atoms and consider entropy, we see that time reversal also seems to leave physical situations and laws unaltered. But is it really a true symmetry of nature? (In changing all values of the time coordinate with –t, we are *not* saying that time is running backwards. It just means that if we substitute –t for t in every physical law, they are still valid. Whether time actually has a predetermined direction, determined by something other than increasing entropy is perhaps unknowable.)

p. 184-6: I don't understand exactly why, but the discussion abruptly turns to antimatter and the C symmetry. Maybe that is because the experiment(s) that show the failure of T also involve antimatter (which involves C). Anyway, this section is about experiments that show that CP is not a symmetry of nature, in the case of the weak interaction. This means that simultaneously reversing all charges (C) and reflecting in space (P) changes the behavior of the weak interaction. (This incompleteness of the CP symmetry is apparently needed to allow a small difference in original matter and antimatter quantities so that there is a little matter left over. How this happens is an important theoretical frontier.)

It isn't explained here, but the final situation on C, P, and T is that each reflection alone, or any pair done together, does not give a symmetry of nature.

p. 187-8: In quantum mechanics as well as in other situations, we would really like it if the sum of the probabilities of all possible outcomes must always be exactly 1. From this (very well supported) assumption, it has been proven theoretically that the combined CPT symmetry (reflecting all 3 simultaneously) must be an exact symmetry of nature. But the sum of probabilities is a human postulate that nature might possibly ignore. Again, experimental verification is needed. So far it looks good.

Chapter 9, Broken Symmetry

p. 189-90: First, an attempt to clarify terminology. I think "broken symmetry" and "hidden symmetry" refer to the same thing from different perspectives. When a symmetry breaks, something that could be any of a number of different ways gets locked into one of them. After the symmetry breaking event, if you observe it in the state that resulted arbitrarily, you may not realize that it had previously been unbroken (i.e. capable of being other ways), so the symmetry is hidden.

P. 191-2: In physics, the unbroken state of a symmetrical object is often higher energy than the states that may result after the symmetry is broken. It may be kept in the symmetrical state because there is a lot of energy around, such as in the high temperature after the big bang. These symmetrical states may break when the universe cools. Much insight into physics has come from noticing that some particle has some similarity to others, and trying to figure out why and if they are different outcomes of the same broken symmetry.

P. 192-7: Magnetism is an illustration of the concept of symmetry. Enough said.

p. 198: The whole concept of the Mexican hat potential and the symmetry breaking of rolling off the peak was originally conceived to explain something about the Higgs field, and is often called the "Higgs mechanism". The idea is that the potential energy is higher at the peak than the brim, so eventually, things will change (the symmetry breaks, maybe because the universe cools) and the state of the system changes to a lower energy one in the brim. Because the unbroken situation was inherently unstable, this is called spontaneous symmetry breaking.

p. 200: A particularly interesting problem of modern physics is inflation. One hypothesis about inflation is that it is associated with the breaking of a symmetry of the inflaton field. In this hypothesis, the inflaton field has a higher energy in its initial state (the peak of the "Mexican hat") than a later state (the brim). The process of rolling off the peak into the brim could be seen as a spontaneous breaking of a symmetry of the inflaton field. Because the brim has lower energy, the energy previously stored in the field becomes available for other things. In the case of inflation, that sudden release of energy becomes the big bang. There are lots of variations on this idea.

Chapter 10 Quantum Mechanics

P. 215-6 and note 5: The Heisenberg uncertainty principle. He doesn't say it, but this is not obscure magic performed by nature. This is just a normal mathematical consequence of the use of complex exponentials (sinusoids) to model wave functions. See http://en.wikipedia.org/wiki/Uncertainty_principle#Introduction, which contains

> Mathematically, in wave mechanics, the uncertainty relation between position and momentum arises because the expressions of the wavefunction in the two corresponding bases are Fourier transforms of one another (i.e., position and momentum are conjugate variables). A nonzero function and its Fourier transform cannot both be sharply localized.

The fact that nature actually behaves that way lends credence to the model, and perhaps to the ultimate affinity of nature and mathematics.

P. 216-226 presents a clear overview of the nature of the quantum mechanical wave equation (WF, see note 1) and its behavior in bound particle states, and quantization of angular momentum. P. 218-20: Born's interpretation of the magnitude squared of the wave function as the probability density of a particle's presence allows the idea that the wave function can have non-zero values in different places while the electron can only actually be found in one. P.220-4: A bound state means that the physical location of a particle is restricted. Bound states give rise to quantized energy states. By the uncertainty principle, the more narrow the restriction (less uncertainty of position), the greater the uncertainty in its momentum (which means its momentum can be larger, to accommodate the uncertainty). P.226: Bosons and fermions, odd or even multiples of hbar/2, etc. Not explicitly stated (almost, on p.230) is that any atom with an even number of N+P+e (all spin +- ½) must be a boson, because the sum of an even number of +- ½ must be an integer. Likewise, odd => fermion.

P. 227-31: Symmetry of identical particles allows both + and – amplitude solutions for any two identical particle wave equation because only the squared magnitude (probability density and all other observables) must be the same when swapping them. He doesn't say why, but boson wave functions have same sign when swapping identical particles, and fermions have opposite sign. Since the only way to have opposite sign amplitude and same mag squared is for magnitude = 0, (identical) fermions must have zero probability to occupy the same space, which is the Pauli exclusion principle. Since boson WFs can have non-zero amplitude for identical particles in the same place, they can pile up to make super fluids and lasers.

P. 233-6: Similarly, + and – energy solutions to the electron WF are allowed. Dirac's insight and unconventional interpretation of this odd fact allowed him to predict antimatter.

Chapter 11 The Hidden Symmetry of Light

This chapter again becomes confusing, at least in part because of insufficient specificity. I'll try to parse it as well as I can.

P. 237-9: Charge conservation is absolute, therefore there must be a continuous symmetry associated with it. What might that be?

P. 239-40: The concept of a gauge field is introduced quite vaguely. The example used (apparently the same one to be used later as the photon field?) is a field that determines the electromagnetic field, but an unspecified infinite family of gauge fields (related by some continuous transformation) produce the same EM field. Thus, the combined EM/gauge field has a symmetry in that various forms of the gauge field produce the same EM field, and therefore the same observable situation. The gauge field is not independently observable. So if we take this to be a hidden symmetry, we need a conserved quantity to go with it. Charge is nominated, for no apparent reason. This concept is called gauge invariance, gauge symmetry, etc.

So now we have two unobservable features (symmetries) in electromagnetism: The phase part of electron wave function, and the gauge field discussed above. Can these be combined?

P. 241-2: The concept of electron phase is rather laboriously introduced. Both the momentum and energy of an electron are encoded by its phase behavior. Energy is determined by frequency, which is rate of change of phase vs. time; momentum by wavelength, which is rate of change of phase vs. distance. However, phase itself is unobservable. If we change the phase of an electron by the same amount everywhere, its behavior is unchanged. This is global phase symmetry. But if we want to allow phase to change randomly in space and time (p. 242 bottom, see also note 3), a local phase transformation, we have also applied changes to the electron's energy and momentum, so this is not a valid symmetry.

P. 243-6: To make this local transformation into a local symmetry, we need to introduce something else. That is our gauge field. It's job is to "compensate" the energy and momentum changes caused by the local variations in phase (as well as whatever else is needed – see note 4). It turns out that this new gauge field describes the photon. (Is the electron phase part of the variable part of the gauge field? The idea that the electron/photon/gauge fields are to be thought of as a single entity is new to me. Maybe a review of Schumm would help here, or another QFT book.)

P. 254-5: All known forces are based on gauge symmetry theories. (Is the coordinate system invariance of GR a gauge symmetry? How?) The other 3 are Yang-Mills theories. What is the core idea of that?

http://en.wikipedia.org/wiki/Gauge_theory

http://en.wikipedia.org/wiki/Yang%E2%80%93Mills_theory http://en.wikipedia.org/wiki/Lagrangian

Chapter 12 Quarks and Leptons

P. 262: The "particle zoo" of the 1950s, now known to be various kinds of baryons (3 quarks) and mesons (2 quarks). Very confusing, but with patterns of similarity among them. Gell-Mann was able to classify them by finding symmetries in the patterns of their properties. His symmetries may not be terribly useful now, but the breakthrough was in showing that finding symmetries was the doorway to understanding the nature of the new particles.

P. 263-9: A pretty good sketch of the standard model, at a level appropriate to the subject of symmetry. Be sure to read notes 5-8. P. 267-8: The three generations of particles. The first is the lightest; there is nothing lighter for them to decay into, so they are what make up ordinary matter. Apparently 3 are needed to allow an excess of matter over antimatter (note 6). It looks like there may be no more than 3, because the top quark mass is almost the same as the "Fermi scale" mass (so coupling constant is near 1, p. 282), and possibly that's the largest possible mass.

P. 270-6: The symmetry of the strong interaction and quark colors. The properties of the new particles require a symmetry of 3 "colors" of quarks. This can be modeled as a 3-complex-dimensional space for the quark wave function where the wave function is a vector pointing in any direction in that space. Just as the electron phase symmetry is a (part of a?) gauge symmetry that requires the existence of the photon field, this higher dimensional symmetry of quarks is a gauge symmetry that requires new particles to make sense. In this case, 8 gluons are needed, and QCD is born. In addition to exchanging quark colors, gluons also transfer energy and momentum. Quark confinement, quark jets.

P. 276-80: The gauge symmetry of the (electro)weak force. Here, we have a symmetry between the upper and lower members of a weak doublet, e.g. electron/electron-neutrino, up/down quark, etc. This is represented by a wave function vector in complex 2-space. As before, to preserve the symmetry of this direction, more fields/particles are needed, in this case W+- and Z. Because these are closely related to the photon, we ultimately find that the electromagnetic and weak forces are unified, and there is a symmetry among all four gauge bosons. The W and Z are very heavy, therefore unlikely to appear spontaneously from the vacuum, or persist for long, so the weak force is weak and short-ranged. The photon is massless, so the EM force is long-ranged. So how can there be a symmetry among such different particles?

P. 280-3: Very sketchy. The symmetry is broken by the breaking of the Higgs field symmetry, which happens because the field has non-zero value at its lowest energy state (p. 201). The W and Z interact with this non-zero Higgs field, but the photon does not, so their symmetry is broken. Mass apparently arises from the coupling strength to this field, with a coupling constant for each particle that scales down its mass from the Fermi scale mass. For the top quark, this constant is nearly 1, which suggests that it may be the most massive particle.

There may be a subtle difference in how the Higgs field gives mass to W and Z and the other particles, but it is not mentioned. Compare, from three successive paragraphs on p. 282-3: "…no theory for the origin of the coupling constants…", "…precisely predicted the coupling strength…", and "…completely controls the mass generation…". Maybe he means that the masses of W and Z are predicted by theory, other particles are not. I think there's something left out.

P. 284-7: Here we have some disciplined speculation about future theoretical developments. At the very least, gravity is unaccounted for in the standard model. We also need an explanation for dark matter and dark energy. The present expectation is that the standard model is part of a larger structure, which includes the Higgs boson and gravity. Experiment presently only shows one Higgs boson, although various theories suggest there are more. Extra dimensions that make some sense in Einstein's equation for gravity support the same idea in string theory. Supersymmetry (AKA supersymmetric string theory) draws on that, plus the idea that each known particle has a supersymmetric partner (fermions to bosons and vice versa). This could again be a gauge symmetry. It may be associated with the Fermi scale energy. (175 GeV? Wikipedia says 246 GeV is the value of the Higgs vacuum expectation value. This seems to be the same thing, defined differently by sqrt(2). See http://en.wikipedia.org/wiki/Top_quark#Mass_and_coupling_to_the_Higgs_boson. m_t is mass of top quark.) In that case, we may be close to able to observe it. The lightest supersymmetrical particle(s) might be stable, and make up dark matter. String theory may explain gravity, since the lowest string vibration mode looks like a graviton.

Appendix

P. 295-306: An introduction to discrete groups, with brief discussion of commutativity. Not bad, but we are mainly interested in continuous groups.

P. 306-12: A simple example of how symmetry can be used in physical reasoning. Mostly discrete symmetry, but the last paragraph shows that the same kind of reasoning can apply to continuous symmetries.

P. 312-6: An incomplete grab bag of concepts about continuous groups. Some of this is useful for us, some not.