http://sackett.net/SymmetryInParticlePhysics.pdf 11/16/2020

Cosmology Group,
Here's my contribution to the symmetry and the Standard Model of Particle Physics discussion. It comes from surfing the web so accuracy might not be all that good. I particularly want to build a larger table of symmetries and corresponding conservation laws so if you have additions or corrections to what I have here, please let me know.
Thanks,
Bill Sackett

Noether's Theorem: *For every continuous symmetry of the laws of physics, there must exist a conservation law.*
For every conservation law, there must exist a continuous symmetry.
(For more on Noether, see http://sackett.net/Noether.pdf)

Constant speed of light	<->	Lorentz invariance
Conservation of a relativistic analogue of angular momentum	<->	Boost symmetry *
Conservation of energy	<->	Time shift symmetry
Conservation of charge	<->	Local gauge invariance
Conservation of baryon number	<->	Global gauge invariance
Conservation of momentum	<->	Space translational symmetry
Conservation of angular momentum	<->	Space rotational symmetry
Conservation of lepton number	<->	U(1) symmetry **
Conservation of weak isospin	<->	SU(2) symmetry **
Conservation of mass	<->	Time shift symmetry

(because of Einstein's mass-energy equivalence? Maybe the whole conservation law should be mass-energy because of relativistic speeds and nuclear reactions.)

* The observation that a physics experiment gives the same results in inertial reference frames moving at a constant velocity with respect to each other is called 'boost symmetry'. Lorentz invariance involves both orientation and boost but entries in the table for Lorentz invariance and boost symmetry may be redundant.
From http://kgbudge.com/essays/U1xSU2xSU3_2.html:
The Standard Model of particle physics is based on a combination of three internal symmetries, U(1) X SU(2) X SU(3).
** From http://kgbudge.com/essays/U1xSU2xSU3_4.html:
Noether's Theorem tells us that every continuous symmetry of the Lagrangian density corresponds to some conservation law. We find that the original U(1) symmetry corresponds to the conservation of electric charge, and that the SU(2) symmetry introduces three more U(1) symmetries that correspond to conservation

of electron, muon, and tau lepton numbers. The SU(2) symmetry itself corresponds to conservation of a quantity called "weak isospin." When we promote SU(3) to a local symmetry of the Lagrangian density, we obtain a gauge field for each parameter describing an element of SU(3). There are eight such parameters, and therefore eight such gauge fields. At present, it is thought that there is no spontaneous breaking of the SU(3) symmetry, so these eight gauge fields remain massless. The particles corresponding to these fields are called *gluons*, and they are responsible for the color force that holds quarks together.
[To tie together some of these ideas, the abstract definition of charge at https://en.wikipedia.org/wiki/Charge_(physics) helps.]

All forces come from Local Gauge Symmetries:
This includes electromagnetism (local phase invariance), gravity (local coordinate system invariance), the strong force (associated with quark color, and the freedom to rotate a quark in color space locally), and the weak interactions (involving rotations between charged leptons and neutrinos, or between up quarks and down quarks).

No two identical fermions can occupy the same quantum state. This was first known as the "Pauli Exclusion Principle", after the Austrian-Swiss theorist Wolfgang Pauli. Pauli also proved that it comes from the basic symmetries of the laws of physics, though few theorists understand this proof in detail; it is much harder than the Noether theorem.

Group theory is the mathematical tool that is used to work with symmetries. Here's a quote from Johnson and Richman, Numbers and Symmetry (p 125): "We measure the symmetry of an object by looking at its group of symmetries. Henri Poincaré said, 'Mathematicians do not deal in objects, but in relations between objects; thus they are free to replace some objects by others so long as the relations remain unchanged.' From this point of view we are not so much interested in the symmetries (the objects), as in the relations among the symmetries - how they compose to form an equation like ' = '. That is, we are interested in the *abstract* properties of symmetry groups – the properties of the group operation, rather than the properties of the symmetries themselves.

An abstract group is like a number – a number with structure. We use numbers to measure distance, and we use groups to measure symmetry."

From Frank Wilczek, The Lightness of Being, pages 196-197:

"There's another promising idea about what the dark matter is, which emerges from a different proposal for improving the equations of physics. As we've discussed, QCD [Quantum Chromodynamics] is in a profound and literal sense constructed as the embodiment of symmetry. There is an almost perfect match between the observed properties of quarks and gluons and the most general properties allowed by local color symmetry, in the framework of special relativity and quantum mechanics. The only exception is that the established symmetries of QCD fail to forbid one sort of behavior that is not observed to occur. The established symmetries permit a sort of interaction among gluons that would spoil the symmetry of equations of QCD under a change in the direction of time. Experiments provide severe limits on the possible strength of that interaction. The limits are much more severe than might be expected to arise accidentally.

The Core theory does not explain this 'coincidence.' Roberto Peccei and Helen Quinn found a way to expand the equations that would explain it. Steven Weinberg and I, independently, showed that the expanded equations predict the existence of new, very light, very weakly interacting particles called axions. Axions are also serious candidates to provide the cosmological dark matter." [Axions are bosons theorized to have condensed out of the primordial plasma with zero velocity and low mass but with a density high enough to possibly account for dark matter (Sean Carroll at the end of Dark Side of the Universe, Lecture 13).]

From http://en.wikipedia.org/wiki/Quantum_chromodynamics:

There are two different types of **SU**(3) symmetry: there is the symmetry that acts on the different colors of quarks, and this is an exact gauge symmetry mediated by the gluons, and there is also a flavor symmetry which rotates different flavors of quarks to each other, or *flavor **SU**(3)*. Flavor **SU**(3) is an approximate symmetry of the vacuum of QCD, and is not a fundamental symmetry at all. It is an accidental consequence of the small mass of the three lightest quarks.

From Sean Carroll's YouTube on Gauge Theory:

"W and Z bosons are the physical excitations from the vibrations in the SU(2) gauge field." The rotations are of up to down quarks and electrons to electron neutrinos, … They don't look like symmetries because the symmetry is broken.***

Utiyama described a gauge theory for gravity with SO(3,1) (not SO(4) because you can't rotate space into time), the Lorentz group. Poincare group (Lorentz plus translations) is better to work with because you are not just dealing with internal symmetries as you are in QED, QCD, and the Weak Force, you are actually dealing with transformations of spacetime itself.

*** "The existence of these gauge symmetries implies new fields, the connection fields. And the connection fields give rise to physical particles."

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